PART III. continued.

To which are added,

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A COMPENDIOUS SYSTEM OF Natural Philosophy.

With NOTES

Containing the MATHEMATICAL DEMONSTRATIONS, and some occasional REMARKS.

PART III. continued.

CONTAINING

CATOPTRICS,

AND THE

Doctrine of LIGHT and COLOURS.

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SECOND EDITION.

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CHAP. VIII.

Of the Manner *wherein* Light *is* reflected.

THE *Refraction* of Light has been considered, and explained: its *Reflection* is now to be inquired into *(a)*. And first for the Manner wherein it is performed.

When a Ray of Light falls upon a Body not transparent, *Part* or *all* of it *is reflected*; if any enters, it is suffocated and lost within the Body: When it falls upon a *transparent* Body, *Part* of it *is reflected*, and *Part* enters; of what enters some is also suffocated and lost in the Body; the rest, when it arrives at the *other Side*, some of it is reflected there, the Remainder going out and leaving the Body, unless its Inclination to *that Side* exceeds a *certain* Degree; which if it does, it is *all reflected* there. And the Power whereby a Ray is reflected at *this other Side* of a Body (which for Distinction sake I shall hereafter call *the second Surface*) is stronger than that by which it would be reflected by the same Surface, were it about to enter the Body there with an equal Degree of Obliquity.

(a) The Subject of this *Inquiry* is distinguished from that of the former, by the Name of *Catoptrics*, as tending to explain the Manner in which Objects appear, when seen by *reflected* Light.

The *Degree of Inclination* neceſſary to cauſe a *total Reflection* of a Ray at the ſecond Surface of a *Medium*, is that which requires that the refracted Angle (was the Ray ſuppoſed to paſs out there) ſhould be equal to, or greater than a right one; and conſequently it depends on the refractive Power of the *Medium* through which the Ray paſſes; and is therefore different in different *Media*. When a Ray paſſes through Glaſs ſurrounded with Air, and is inclined to its ſecond Surface under an Angle of 42 *Degrees or more*, it will be *wholly reflected* there *(b)*. For as 11 is to 17,

(b) From hence it follows, that when a Ray of Light arrives at the ſecond Surface of a tranſparent Subſtance with as great or greater Degree of Obliquity, than that which is neceſſary to make a total Reflection, it will there be all returned back to the firſt; and if it proceeds towards that with as great an Obliquity as it did towards the other (which it will do if the Surfaces of the *Medium* be parallel to each other) it will there be all reflected again, &c. and will therefore never get out, but paſs from Side to Side, till it be wholly ſuffocated and loſt within the Body.

From hence may ariſe an obvious Inquiry, how it comes to paſs that Light, falling very obliquely upon a Glaſs-Window from without, ſhould be tranſmitted into the Room? In Anſwer to this, it muſt be conſidered, that however obliquely a Ray falls upon the firſt Surface of any *Medium* whoſe Sides are parallel (as thoſe of the Glaſs in a Window are) it will ſuffer ſuch a Degree of Refraction in entering there, that it ſhall fall upon the ſecond with a leſs Obliquity than that which is neceſſary to cauſe a total Reflection. For Inſtance let the *Medium* be Glaſs, as ſuppoſed in the preſent Caſe, then as 17 is to 11 (the *Ratio* of Refraction out of Air into Glaſs) ſo is the Sine of the largeſt Angle of Incidence with which a Ray can fall upon any Surface, to the Sine of a leſs Angle than that of total Reflection. And therefore, if the Sides of a Glaſs be parallel, the Obliquity, with which a Ray falls upon the firſt Surface, cannot be ſo great, but that it ſhall paſs the ſecond without ſuffering a total Reflection there.

(the

(the *Ratio* of Refraction out of Glaſs into Air) ſo is the Sine of an Angle of 42 Degrees to a fourth Number, that will exceed the Sine of a right Angle.

When Light paſſes out of a denſer into a rarer *Medium*, the nearer the ſecond *Medium* approaches the firſt in Denſity (or more properly in its refractive Power) the leſs of it will be reflected in paſſing from one to the other: and when their refracting Powers are equal, all of it will paſs into the ſecond *Medium.*

Whether Light be reflected from the firſt or ſecond Surface of a Body, the Law it obſerves is this, viz. *That the Angle of Reflection of each Ray ſhall be equal to the Angle of Incidence of the ſame.*

By the *Angle of Reflection* is meant the Angle comprehended between a Perpendicular to the Surface at the Point where the Reflection is made, and the reflected Ray.

Theſe are all the Circumſtances attending the Reflection of Light neceſſary to be taken Notice of at preſent: There are others, but they reſpect the *Doctrine of Light and Colours* not yet explained; we ſhall therefore paſs them by till we treat of that Subject, and in the mean Time proceed to conſider the Reflection of Light from plain and ſpherical Surfaces.

CHAP. IX.

Of the Reflection *of* Light *from* plain *and* ſpherical Surfaces.

IT was obſerved in the foregoing Chapter, that the Law of Reflection is ſuch, that the Angle of Reflection of each Ray ſhall be equal to the Angle of Incidence of the ſame. From whence the ſeven following Propoſitions relating to the Reflection of Light from plain and ſpherical Surfaces may be deduced.

I. Rays of Light reflected from a plain Surface have the ſame Degree of Inclination to each other that their reſpective incident ones have.

For the Angle of Reflection of each Ray being equal to that of its reſpective incident one, it is evident that each reflected Ray will have the ſame Degree of Inclination to that Portion of the Surface from whence it is reflected, that its incident one has; but it is here ſuppoſed that all thoſe Portions of Surface from whence the Rays are reflected are ſituated in the ſame Plain; conſequently the reflected Rays will have the ſame Degree of Inclination to each other that their incident ones have, from whatever Part of the Surface they are reflected.

See

See this and the following Propositions expressed more determinately, and demonstrated in the Note below (a).

II. Parallel Rays, reflected from a concave Surface, are render'd converging.

H To

(a) Proposition I.

Of the Reflection of Rays from a plain Surface.

When Rays fall upon a plain Surface, if they diverge, the Focus *of the reflected Rays will be at the same Distance behind the Surface, that the* Radiant *Point is before it: if they converge, it will be at the same Distance before the Surface, that the imaginary* Focus *of the incident Rays is behind it.*

This Proposition admits of two Cases.

Case I. *Of diverging Rays.*

Dem. Let AB, AC (Fig. 45.) be two diverging Rays incident in the plain Surface DE, the one perpendicularly, the other obliquely; the perpendicular one AB will be reflected to A proceeding as from some Point in the Line AB produced; the oblique one AC will be reflected into some Line as CF such, that the Point G, where the Line FC produced intersects the Line AB produced also, shall be at an equal Distance from the Surface DE with the *Radiant* A. For the Perpendicular CH being drawn, ACH and HCF will be the Angles of Incidence and Reflection, which being equal, their Complements ACB and FCE are so too: but the Angle BCG is equal to FCE as being vertical to it; therefore in the Triangles ABC and GBC the Angles at C are equal, the Side BC is common, and the Angles at B are also equal to each other, as being right ones; therefore the Lines AB and BG, which respect the equal Angles at C, are also equal, and consequently the Point G, the *Focus* of the incident Rays AB, AC, is at the same Distance behind the Surface, that the Point A is before it Q. E. D.

Case II. *Of converging Rays.*

This is the Converse of the former Case. For supposing FC and AB to be two converging incident Rays, CA and BA will be the reflected ones (the Angles of Incidence in the former

Case

To illustrate this, let AF, CD, EB, (Fig. 43.) represent three parallel Rays falling upon the concave Surface FB, whose Center is C. To the Points F and B draw the Lines CF, CB; these being drawn from the Center will be

Case being now the Angles of Reflection, and *vice versâ*) having the Point A for their *Focus*; but this, from what was demonstrated above, is at an equal Distance from the reflecting Surface with the Point G, which in this Case is the imaginary *Focus* of the incident Rays, FC, and AB.

Observat. I. *It is not here, as in the Refraction of Rays in passing through a plain Surface, where some of the refracted Rays proceed as from one Point, and some as from another* (*See Observat.* I. *Chap.* 3. *in the Notes*) *but they all proceed after Reflection as from one and the same Point, however obliquely they may fall upon the Surface; for what is here demonstrated of the Ray AC holds equally of any other, as AI, AK, &c.*

The Case of parallel Rays incident on a plain Surface, is included in this Proposition; for in that Case we are to suppose the *Radiant* to be at an infinite Distance from the Surface, and then by the Proposition, the *Focus* of the reflected Rays will be so too: that is, *the Rays will be parallel after Reflection, as they were before.*

Proposition II.

Of the Reflection of parallel Rays from a spherical Surface.

When parallel Rays are incident upon a spherical Surface, the Focus *of the reflected Rays will be the middle Point between the Center of Convexity and the Surface.*

This Proposition admits of two Cases.

Case I. *Of parallel Rays falling upon a convex Surface.*

Dem. Let AB, DH, (Fig. 46) represent two parallel Rays incident on the convex Surface BH, the one perpendicularly, the other obliquely; and let C be the Center of Convexity; suppose HE to be the reflected Ray of the oblique Incident one DH proceeding as from F a Point in the Line AB produced. Through the Point H draw the Line CI, which will be perpendicular to the Surface at that Point, and the Angles DHI and IHF, *being the Angles and Incidence and Reflection*, will be equal. To

be perpendicular to the Surface at those Points. The incident Ray CD also passing through the Center will be perpendicular to the Surface, and therefore will return after Reflection in the same Line; but the oblique Rays AF

 and

To the former of these, the Angle HCF is equal, *the Lines AC and DH being parallel*, and to the latter the Angle CHF *as being vertical*; wherefore the Triangle CFH is *Isosceles*; and consequently the Sides CF and FH are equal: But supposing BH to *vanish*, FH is equal to FB, and therefore upon this Supposition FC and FB are equal, that is, the *Focus* of the reflected Rays is the middle Point between the Center of Convexity and the Surface. Q. E. D.

Case II. *Of parallel Rays falling upon a concave Surface.*

Dem. Let AB, DH (Fig. 47.) be two parallel Rays incident, the one perpendicularly, the other obliquely, on the concave Surface BH, whose Center of Concavity is C. Let BF and HF be the reflected Rays meeting each other in F: this will be the middle Point between B and C. For drawing through C the Perpendicular CH, the Angles DHC and FHC, *being the Angles of Incidence and Reflection*, will be equal, to the former of which the Angle HCF is equal, *as alternate*; and therefore the Triangle CFH is *Isosceles*. Wherefore CF and FH are equal: but if we suppose BH to *vanish*, FB and FH are also equal, and therefore CF is equal to FB; that is, the focal Distance of the reflected Rays is the middle Point between the Center and the Surface. Q. E. D.

Observat. II. *It is here observable, that the farther the Line DH, either in Figure the* 46 *or* 47, *is taken from AB, the nearer the Point F falls to the Surface. For the farther the Point H recedes from B, the larger the Triangle CFH will become; and consequently since it is always an* Isosceles *one, and the Base CH, being the* Radius, *is every where of the same Length, the equal Legs CF and FH will lengthen; but CF cannot grow longer unless the Point F approach towards the Surface. And the farther H is removed from B, the faster F approaches to it.*

This is the Reason, that whenever parallel Rays are considered, as reflected from a spherical Surface, the Distance of the oblique one from the perpendicular one is taken so small with respect to the focal Distance of that Surface, that without any physical *Error it may be supposed to* vanish.

and EB will be reflected into the Lines FM and BM situated on the contrary side their respective Perpendiculars CF and CB. They will therefore proceed converging after Reflection towards some Point as M, in the Line CD: which Point, by what is demonstrated in the Note last referred to, will be in the Middle between C and D.

III.

From hence it follows, that if a Number of parallel Rays as AB, CD, EG, &c. fall upon a convex Surface, as expressed Figure the 48, *and if BA, DK, the reflected Rays of the incident ones AB, CD, proceed as from the Point F, those of the incident ones CD, EG,* viz. *DK, GL, will proceed as from N, those of the incident ones EG, HI, as from O, &c. because the farther the incident ones CD, EG &c. are from AB, the nearer to the Surface are the Points F, f, f, in the Line BF, from which they proceed after Reflection; so that properly the* Foci *of the reflected Rays BA, DK, GL, &c. are not in the Line AB produced, but in a curve Line passing through the Points F, N, O, &c.*

The same is applicable to the Case of parallel Rays reflected from a concave Surface, as expressed by the pricked Lines on the other half of the Figure, where PQ, RS, TV, are the incident Rays; QF Sf, Vf, the reflected ones intersecting each other in the Points X, Y, and F; so that the Foci *of those Rays are not in the Line FB, but in a Curve passing through those Points.*

Had the Surface BH in Figure 46, *or* 47, *been form'd by the Revolution of a* Parabola *about its* Axis *having its* Focus *in the Point F, all the Rays reflected from the convex Surface would have proceeded as from the Point F, and those reflected from the concave would have fallen upon it, however distant their incident ones AB, DH, might have been from each other. For in the* Parabola, *all Lines drawn parallel to the* Axis *make Angles with the Tangents to the Points where they cut the* Parabola *(that is, with the Surface of the* Parabola) *equal to those which are made with the same Tangents by Lines drawn from thence to the* Focus. *(De* L'Hospital *Sections Coniques. Liv.* I. *Prop.* 5.) *Therefore, if the incident Rays describe those parallel Lines, the reflected ones will necessarily describe these other, and so will all proceed as from, or meet in, the same Point.*

III. Converging Rays falling on the like Surface are made to converge more.

For every thing remaining as above, let GF, HB, be the incident Rays. Now because these Rays have larger Angles of Incidence than the parallel ones AF and EB in the foregoing Case, their Angles of Reflection will also be larger than theirs; they will therefore converge after Reflection, suppose in the Lines FN, and BN, having their Point of Con-

Proposition III.

Of the Reflection of diverging and converging Rays from a spherical Surface.

When Rays fall upon any spherical Surface, if they diverge, the Distance of the Focus *of the reflected Rays from the Surface is to the Distance of the* Radiant *Point from the same (or, if they converge, to that of the imaginary* Focus *of the incident Rays) as the Distance of the* Focus *of the reflected Rays from the Center is to the Distance of the* Radiant *Point (or imaginary* Focus *of the incident Rays) from the same.*

This Proposition admits of ten Cases.

Case I. *Of diverging Rays falling upon a convex Surface.*

Dem. Let RB, RD (Fig. 49) represent two diverging Rays flowing from the Point R as from a *Radiant,* and falling the one perpendicularly, the other obliquely, on the convex Surface BD, whose Center is C. Let DE be the reflected Ray of the incident one RD, produce ED to F, and through R draw the Line RH parallel to FE till it meets CD produced in H. Then with the Angle RHD be equal to EDH the Angle of Reflection, *as being alternate to it,* and therefore equal also to RDH which is the Angle of Incidence; wherefore the Triangle DRH is *Isosceles,* and consequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are similar, (or to express it in *Euclid*'s Way, the Sides of the Triangle RHC are cut proportionably. 2. Elem. 6.) and therefore FD is to

Concourse N farther from C than the Point M; that to which the parallel Rays AF and EB converged to in the foregoing Case: and their precise Degree of Convergency as determined in the Note, will be greater than that wherein they converged before Reflection.

IV. Diverging Rays, falling upon the like Surface, are after Reflection parallel, diverging, or converging. If they diverge from the *Focus* of parallel Rays, they then become parallel; if from

to RH, or its equal RD, as CF to CR; but BD *vanishing*, FD and RD differ not from FB and RB, wherefore FB is to RB also, as CF to CR; that is, the Distance of the *Focus* from the Surface is to the Distance of the *Radiant* Point from the same, as the Distance of the *Focus* from the Center is to the Distance of the *Radiant* from thence. Q. E. D.

Case II. *Of converging Rays falling upon a concave Surface.*

Dem. Let KD and CB be the converging incident Rays having their imaginary *Focus* in the Point R, which was the *Radiant* in the foregoing Case. Then as RD was in that Case reflected into DE, KD will in this be reflected into DF; for, since the Angles of Incidence in both Cases are equal, *as they are by being vertical*, the Angles of Reflection will be so too; so that F will be the *Focus* of the reflected Rays: but it was there demonstrated that FB is to RB as CF to CR, that is, the Distance of the *Focus* from the Surface is to the Distance (in this Case) of the imaginary *Focus* of the incident Rays, as the Distance of the *Focus* from the Center is to the Distance of the imaginary *Focus* of the incident Rays from the same. Q. E. D.

Case III. *Of converging Rays falling upon a convex Surface, and tending to a Point between the* Focus *of Parallel Rays and the Center.*

Dem. Let BD (Fig. 50.) represent a convex Surface whose Center is C, and *Focus* of parallel Rays is P; and let AB, KD, be two converging Rays incident upon it, and having their imaginary *Focus* at R, a Point between P and C. Now because KD tends

from a Point nearer to the Surface than that, they will diverge, but in a less Degree than before Reflection; if from a Point between that and the Center, they will converge after Reflection and that to some Point on the contrary Side the Center, but situated farther from it than the Point they diverged from: if the incident Rays diverge from a Point beyond the Center, the reflected ones will converge to one on the other Side of it, but nearer to it than the

tends to a Point between the *Focus* of parallel Rays and the Center, the reflected Ray DE will diverge from some Point on the other Side the Center, suppose F; as explained above in the Text under Proposition the VIIth. Through D draw the Perpendicular CD and produce it to H, then will KDH and HDE be the Angles of Incidence and Reflection, which being equal, their vertical ones RDC and CDF will be so too, and therefore the *Vertex* of the Triangle RDF is bisected by the Line DC: wherefore (3 El. 6.) FD and DR, or, BD vanishing, FB and BR are to each other as FC to CR; that is, the Distance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident ones, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

Case IV. *Of diverging Rays falling upon a concave Surface and proceeding from a Point between the* Focus *of parallel Rays and the Center.*

Dem. Let RB, RD, be the diverging Rays incident upon the concave Surface BD, having their *Radiant* in the Point R, the imaginary *Focus* of the incident Rays in the foregoing Case. Then as KD was in that Case reflected into DE, RD will now be reflected into DF. But it was there demonstrated that FB and RB are to each other as CF to CR; that is, the Distance of the *Focus* is to that of the *Radiant*, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

The Angles of Incidence and Reflection being equal, it is evident, that if in any Case the reflected Ray be made the incident one, *the*

the Point they diverged from; and if they diverge from the Center, they will be reflected thither again.

1. Let them diverge in the Lines MF, MB, proceeding from M the *Focus* of parallel Rays; then as the parallel Rays AF and EB were reflected into the Lines FM and BM (Proposit 2.) these Rays will now on the contrary be reflected into them.

2. Let them diverge from N a Point nearer to the Surface than the *Focus* of parallel Rays, they will then be reflected into the diverging Lines

the incident will become the reflected one: and therefore the four following Cases may be considered respectively as the Converse of the four foregoing; for in each of them the incident Rays are supposed to coincide with the reflected ones in the other. Or they may be demonstrated independently of them as follows.

Case V. *Of converging Rays falling upon a convex Surface, and tending to a Point nearer the Surface than the* Focus *of parallel Rays.*

Dem. Let ED, RB (Fig. 49.) be the converging Rays incident upon the convex Surface BD whose Center is C, and *Focus* of parallel Rays is at P; and let the imaginary *Focus* of the incident Rays be at F, a Point between P and B, and let DR be the reflected Ray. From C and R draw the Lines CH, RH, the one passing through D, the other parallel to FE. Then will the Angle RHD be equal to HDE the Angle of Incidence, *as alternate to it*, and therefore equal to HDR, the Angle of Reflection; wherefore the Triangle HDR is *Isosceles*, and consequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are similar, and therefore RH, or RD, is to FD as CR to CF; but BD vanishing, RD and FD coincide with RB and FB, wherefore RB is to FB as CR to CF; that is, the Distance of the *Focus* from the Surface is to the Distance of the imaginary *Focus* of the incident Rays, as the Distance of the *Focus* from the Center is to the Distance of the imaginary *Focus* of the incident Rays from the same. Q. E. D.

Case

Lines FG and BH which the incident Rays GF and HB deſcribed, that were ſhewn to be reflected into them in the foregoing Propoſition : but the Degree wherein they diverge, as demonſtrated in the Note, will be leſs than that wherein they diverged before Reflection.

3. Let them proceed diverging from X a Point between the *Focus* of parallel Rays and the Center, they then make leſs Angles of Incidence than the Rays MF and MB which became parallel by Reflection, they will conſequently have leſs Angles of Reflection, and proceed therefore converging towards ſome Point as Y; which Point will always fall on the contrary Side the Center, becauſe a re-

I flected

Caſe VI. *Of diverging Rays falling upon a concave Surface, and proceeding from a Point between the* Focus *of parallel Rays and the Surface.*

Dem. Let FD and FB repreſent two diverging Rays flowing from the Point F as a *Radiant,* which was the imaginary *Focus* of the incident Rays in the foregoing Caſe. Then as ED was in that Caſe reflected into DR, FD will be reflected into DK, (for the Reaſon mention'd in Caſe the ſecond) ſo that the reflected Ray will proceed as from the Point R: but it was demonſtrated in the Caſe immediately before-going, that RB is to FB as CR to CF; that is, the Diſtance of the *Focus* from the Surface is to that of the *Radiant* from the ſame, as the Diſtance of the former from the Center is to that of the latter from the ſame. Q. E. D.

Caſe VII. *Of converging Rays falling upon a convex Surface, and tending towards a Point beyond the Center.*

Dem. Let AB, ED, (Fig. 50.) be the incident Rays tending to F, a Point beyond the Center C, and let DK be the reflected Ray of the incident one ED. Then becauſe the incident Ray ED

flected Ray always falls on the contrary Side the Perpendicular with respect to that on which its incident one falls; and, as demonstrated in the Note, it will, be farther distant from the Center than X.

4. If the incident ones diverge from Y, they will after Reflection converge to X, those which were the incident Rays in the former Case being the reflected ones in this.

And lastly, if the incident Rays proceed from the Center, they fall in with their respective Perpendiculars, and for that Reason are reflected thither again.

V. Parallel Rays reflected from a convex Surface are rendered diverging.

To shew this, let AB, GD, EF, (Fig. 44.) be three parallel Rays falling upon the convex Sur-

ED tends to a Point beyond the Center, the reflected Ray DK will proceed as from one on the contrary Side, suppose R; as explained in the Text under Proposition the VIIth. Through D draw the Perpendicular CD and produce it to H. Then will EDH and HDK be the Angles of Incidence and Reflection, which being equal, their vertical ones CDF and CDR will be so too: consequently the *Vertex* of the Triangle FDR is bisected by the Line CD: wherefore (3 Elem. 6.) RD is to DF or BD vanishing. RB is to BF as RC to CF; that is, the Distance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident Rays, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

Case VIII. *Of diverging Rays falling upon a concave Surface, and proceeding from a Point beyond the Center.*

Dem. Let FB, FD, be the incident Rays having their *Radiant* in F, the imaginary *Focus* of the incident Rays in the foregoing Case. Then as ED was in that Case reflected into DK, FD will now be reflected into DR; so that R will be the *Focus* of the re-

Surface BF whoſe Center of Convexity is C, and let one of them, *viz.* GD, be perpendicular to the Surface: through B, D, and F, the Points of Reflection, draw the Lines CV, CG, and CT, which becauſe they paſs through the Center will be perpendicular to the Surface at thoſe Points. The incident Ray GD being perpendicular to the Surface will return after Reflection in the ſame Line, but the oblique ones AB and EF in the Lines BK and FL ſituated on the contrary Side their reſpective Perpendiculars BV and FT. They will therefore diverge after Reflection as from ſome Point M in the Line GD produced; which Point, as demonſtrated in the Note, will be in the middle between D and C.

reflected Rays. But it was demonſtrated in the foregoing Caſe, that RB is to FB as RC to CF; that is, the Diſtance of the *Focus* of the reflected Rays from the Surface is to the Diſtance of the *Radiant* from the ſame, as the Diſtance of the *Focus* of the reflected Rays from the Center is to the Diſtance of the *Radiant* from thence. Q. E. D.

The two remaining Caſes may be conſidered, as the Converſe of thoſe under Propoſition the ſecond of this Note, becauſe the incident Rays in theſe are the reflected ones in them; or they may be demonſtrated in the ſame Manner with the foregoing, as follows.

Caſe IX. *Converging Rays falling upon a convex Surface, and tending to the* Focus *of parallel Rays, become parallel after Reflection.*

Dem. Let ED, RB, (Fig. 49.) repreſent two converging Rays incident on the convex Surface BD, and tending towards F, which we will *now* ſuppoſe to be the *Focus* of parallel Rays; and let DR be the reflected Ray, and C the Center of Convexity of the reflecting Surface. Through C draw the Line CD, and produce

VI. Diverging Rays reflected from the like Surface are rendered more diverging.

For, every thing remaining as above, let GB, GF, be the incident Rays. These having larger Angles of Incidence than the parallel ones AB and EF in the preceding Case, their Angles of Reflection will also be larger than theirs; they will therefore diverge after Reflection, suppose in the Lines BP and FQ, as from some Point N farther from C than the Point M; and the Degree wherein they will diverge, as determined in the Note, will exceed that wherein they diverged before Reflection.

VII.

it to H, drawing RH parallel to ED produced to F. Now it has been demonstrated (Case 5.) where the incident Rays are supposed to tend to the Point F, that RB is to FB as RC to CF; but F in this Case being supposed to be the *Focus* of parallel Rays, it is the middle Point between C and B (by Proposition 2d) and therefore FB and FC are equal, and consequently the two other Terms in the Proportion, *viz.* RB and RC, must be so too; which can only be upon a Supposition that R is at an infinite Distance from B; that is, that the reflected Rays BR and DR be parallel. Q. E. D.

Case X. *Diverging Rays falling upon a concave Surface, and proceeding from the* Focus *of parallel Rays, become parallel after Reflection.*

Dem. Let RD, RB (Fig. 50) be two diverging Rays incident upon the concave Surface BD, as supposed in Case the fourth: where it was demonstrated that FB is to RB as CF to CR. But in the present Case RB and CR are equal, because R is supposed to be the *Focus* of parallel Rays; therefore FB and FC are so too. Which cannot be unless F be taken at an infinite Distance from B; that is, unless the reflected Rays BF and DF be parallel. Q. E. D.

Observat.

VII. Converging Rays reflected from the like Surface, are parallel, converging or diverging. If they tend towards the *Focus* of parallel Rays, they then become parallel; if to a Point nearer the Surface than that, they converge, but in a leſs Degree than before Reflection; if to a Point between that and the Center, they ſhall diverge after Reflection, as from ſome Point on the contrary Side the Center, but ſituated farther from it than the Point they converged to; if the incident Rays converge to a Point beyond the Center, the reflected ones will diverge as from one on the contrary Side of it, but nearer to it than the Point the incident ones converged to: and if the incident Rays converge towards the Center, the reflected ones will proceed as from thence.

1. Let

Obſervat 3. *It is here obſervable, that in the Caſe of diverging Rays falling upon a convex Surface (ſee Fig. 49.) the farther the Point D is taken from B, the nearer the Point F, the* Focus *of the reflected Rays, approaches to B, while the* Radiant *R remains the ſame. For it is evident from the Curvature of a Circle that the Point D (See Fig. 51.) may be taken ſo far from B, that the reflected Ray DE ſhall proceed as from F, G, H, or even from B, or from any Point between B and R, and the farther it is taken from B, the* faſter *the Point, from which it proceeds, approaches towards R: as will eaſily appear if we draw ſeveral incident Rays with their reſpective reflected ones, in ſuch Manner that the Angles of Reflection may be all equal to their reſpective Angles of Incidence, as is done in the Figure. The like is applicable to any of the other* Caſes

1. Let them converge in the Lines KB and LF tending towards M the *Focus* of parallel Rays; then as the parallel Rays AB, EF were reflected into the Lines BK and FL (*Proposit.* 5.) those Rays will now on the contrary be reflected into them.

2. Let them converge in the Lines PB, QF, tending towards N a Point nearer the Surface than the *Focus* of parallel Rays, they will then be reflected into the converging Lines BG and FG, in which the Rays GB, GF, proceeded that were shewn to be reflected into them

Cases of diverging or converging Rays incident upon a spherical Surface. This is the Reason that, when Rays are considered as reflected from a spherical Surface, the Distance of the oblique Rays from the perpendicular one is taken so small, that it may be supposed to vanish.

From hence it follows, that if a Number of diverging Rays are incident upon the convex Surface BD at the several Points B, D, D, &c. *they shall not proceed after Reflection as from any one Point in the Line RB produced, but as from a curve Line passing through the several Points F, f, f,* &c. *The same is applicable in all the other Cases.*

Had the Curvature BD (Fig. 49.*) been* Hyperbolical *having its* Foci *in R and F, then R being the* Radiant *(or the imaginary* Focus *of incident Rays) F would have been the* Focus *of the reflected ones, and* vice versâ, *however distant the Points B and D might be taken from each other. In like Manner had the Curve BD (Fig.* 50.*) been* Elliptical *having its* Foci *in F and R, the one of these being made the* Radiant *(or imaginary* Focus *of incident Rays) the other would have been the* Focus *of the reflected ones, and* vice versâ. *For both in the* Hyperbola *and* Ellipsis, *Lines drawn from each of their* Foci *through any Point make equal Angles with the Tangent to that Point; (De* L'Hospital *Sections Coniques, Liv.* II. *Prop.* 8. *& Liv.* III. *Prop.* 11.*) Therefore, if the incident Rays proceed to or from one of their* Foci, *the reflected ones*

them in the Propoſition immediately foregoing: but the Degree wherein they will converge, as demonſtrated in the Note, will be leſs than that wherein they converged before Reflection.

3. Let them converge in the Lines RB and SF proceeding towards X, a Point between the *Focus* of parallel Rays and the Center; their Angles of Incidence will then be leſs than thoſe of the Rays KB and LF which became parallel after Reflection, their Angles of

ones will all proceed, as from or to the other. So that in order that diverging or converging Rays may be accurately reflected to or from a Point, the reflecting Surface muſt be formed by the Revolution of an Hyperbola *about its longer* Axis, *when the incident Rays are ſuch that their* Radiant, *or imaginary* Focus *of incident Rays, ſhall fall on one Side the Surface, and the* Focus *of the reflected ones on the other: When they are both to fall on the* ſame *Side, it muſt be formed by the Revolution of an* Ellipſis *about its longer* Axis. *However upon Account of the great Facility with which ſpherical Surfaces are formed in Compariſon of that with which Surfaces, formed by the Revolution of any of the* Conic *Sections about their* Axes, *are made, the latter are very rarely uſed. Add to this another Inconvenience,* viz. *that, the* Foci *of theſe Curves being Mathematical Points, it is but one Point of the Surface of an Object that can be placed in any of them at a Time, ſo that it is only* in Theory *that Surfaces formed by the Revolution of theſe Curves about their* Axes *render Reflection perfect.*

Now becauſe the focal Diſtance of Rays reflected from a ſpherical Surface cannot be found by the *Analogy* laid down in the third Propoſition of this Note, without making uſe of the Quantity ſought; I ſhall here give an Inſtance whereby the Method of doing it in all others will readily appear.

Prob. Let it be required to find the focal Diſtance of diverging Rays incident upon a convex Surface, whoſe *Radius* of Convexity is

of Reflection will therefore be less, on which Account they must necessarily diverge, suppose in the Lines BH and FI, from some Point as Y; which Point for the Reason given under Proposition the fourth will fall on the contrary Side the Center with respect to X, and, as demonstrated in the Note, will be farther from it than that.

4. If the incident Rays tend towards Y, the reflected ones will diverge as from X, those, which

is 5 Parts, and the Distance of the *Radiant* from the Surface is 20.

Sol. Call the focal Distance sought x, then will the Distance of the *Focus* from the Center be $5 - x$, and that of the *Radiant* from the same 25; therefore by Proposition the third, we have the following Proportion, *viz.* $x : 20 :: 5 - x : 25$, and, multiplying Extreams together and Means together, we have $25x = 100 - 20x$, which after due Reduction gives $x = \frac{100}{45}$.

If in any Case it should happen, that the Value of x should be a negative Quantity, the focal Point must then be taken on the contrary Side the Surface to that on which it was supposed that it would fall in stating the *Problem*.

If Letters instead of Figures had been made use of in the foregoing Solution a general *Theorem* might have been raised, to have determined the focal Distance of reflected Rays in all Cases whatever. See this done by *Dr. Brown* in his Supplement to *Gregory*'s Optics, pag. 112. Edit. Second.

Because it was observed (Chap. III. in the Notes) that different incident Rays, though tending to or from one Point, would after *Refraction* proceed to or from different Points, a Method was there inserted of determining the *distinct* Point, which each separate Ray entring a spherical Surface converges to or diverges from after *Refraction*: the same has been observed here with regard to Rays *reflected* from a spherical Surface, (See *Observat.* 2 & 3.) but

which were the incident ones in one Caſe, being the reflected ones in the other.

And laſtly, if the incident Rays converge towards the Center, they fall in with their reſpective Perpendiculars; on which Account they proceed after Reflection, as from thence.

but the Method of determining the *diſtinct* Point, to or from which any given incident Ray proceeds after *Reflection*, is much more ſimple. It is only neceſſary to draw the reflected Ray ſuch, that the Angle of Reflection may be equal to the Angle of Incidence, which will determine the Point it proceeds to or from in any Caſe whatever.

Chap. X.

Of the Appearance *of Bodies seen by* Light *reflected from plain and spherical Surfaces.*

IN the Beginning of the seventh Chapter, in which was explained the Appearance of Bodies seen through *refracting* Substances of various Forms, we laid down some Observations respecting the apparent Situation of Bodies seen by *refracted* Light: all which equally respect the apparent Situation of Bodies seen by *Reflection*; to them therefore we refer the Reader. But besides those, there is one peculiar to the Subject of this Chapter, *viz.* *That each Point in the Representation of an Object made by Reflection appears situated somewhere in an infinite right Line that passes through its correspondent Point in the Object, and is perpendicular to the reflecting Surface.*

The Truth of this appears sufficiently from the several Propositions laid down in the foregoing Chapter, in each of which Rays flowing from any *Radiant* are shewn to proceed after Reflection to or from some Point in a Line that passes through the said *Radiant*, and is perpendicular to the reflecting Surface. For Instance (Fig. 43.) Rays flowing from Y are

are collected in X, a Point in the Perpendicular CD, that produced passes through Y; again (Fig. 44.) Rays flowing from G proceed after Reflection, as from N, a Point in the Perpendicular CD, that produced passes through G; and so for the rest [a].

I. When an Object is seen by Reflection from a *plain* Surface, the Image of it appears at the *same Distance* behind the Surface that the Object is placed before it, of the *same Magnitude* therewith, and directly *opposite* to it.

To explain this, let AB (Fig 52.) represent an Object seen by Reflection from the plain Surface SV, and let the Rays AF, AG be so inclined to the Surface that they shall enter an Eye at H after Reflection; and let AE be

[a] This Observation, except where an Object is seen by Reflection from a plain Surface, relates only to those Cases where the Representation is made by means of such Rays, as fall upon the reflecting Surface with a very small Degree of Obliquity; because such, as fall at a considerable Distance from the Perpendicular, proceed not after Reflection as from any Point in that Perpendicular (See the second and third Observations in the Note to the foregoing Chapter) but as from other Points situated in a certain Curve, as there explained; upon which Account these Rays are neglected as forming a confused and deformed Representation. And therefore it is to be remembered, that however the Situation of the Eye, with respect to the Object and reflecting Surface, may be represented in the following Figures, it is to be supposed as situated in such Manner with respect to the Object, that Rays, flowing from thence and entring it after Reflection, may be such only as fall with a very small Degree of Obliquity upon the Surface: that is, the Eye must be supposed to be placed almost directly behind the Object, or between it and the reflecting Surface. The Reason why it is not always so placed, is only to avoid Confusion in the Figures.

perpendicular to the Surface: then by the Observation just laid down, the Point A will appear in some Part of the Line AE produced, suppose I; that is, the oblique Rays AF and AG will proceed after Reflection as from that Point, and further, because the reflected Rays FH, GK, will have the same Degree of Inclination to each other, that their incident ones have (as was shewn in Proposition the first of the foregoing Chapter) that Point must necessarily be at the same Distance from the Surface that the Point A is; the Representation therefore of the Point A, will be at the same Distance behind the Surface, that the Point itself is before it, and directly opposite to it: consequently since the like may be shewn of the Point B, or any other, the whole Image IM will appear at the *same Distance* behind the Surface that the Object is before it, and directly *opposite* to it; and because the Lines AI, BM, which are perpendicular to the plain Surface, are for that Reason parallel to each other, it will also be of the *same Magnitude* therewith. As was to be shewn [b].

II. When an Object is seen by Reflection from a *convex* Surface, its Image appears *nearer* to the Surface, and *less* than the Object.

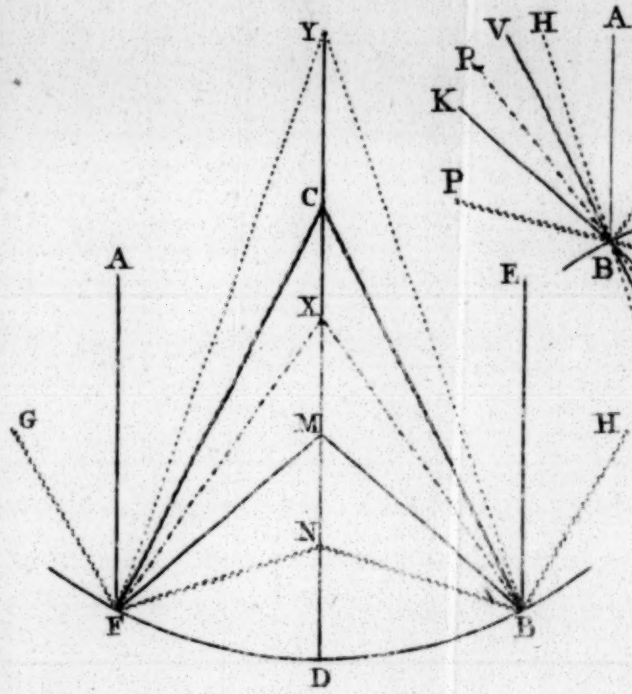
Let

[b] If the Object be placed before a common Looking-glass, and viewed obliquely, three, four, or more Images of it will appear behind the Glass.

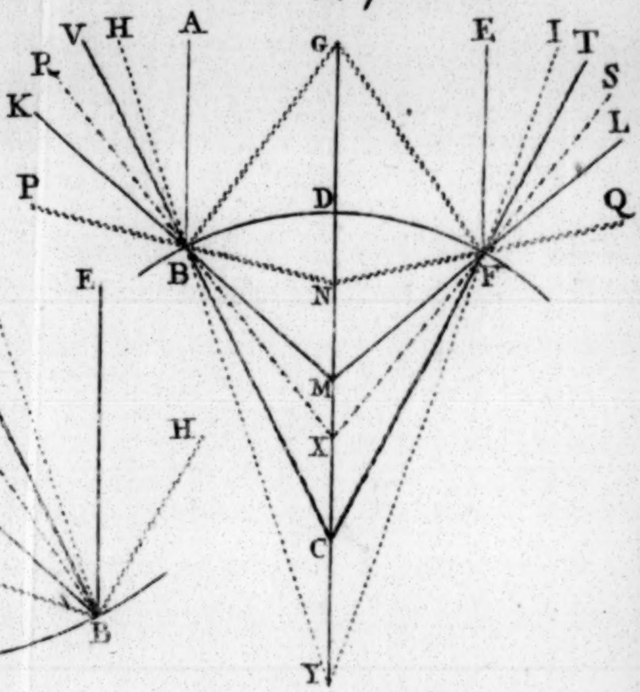
To

Part III. Plate VII. p. 120.

F. 43. p. 102.



F. 44. p. 110.



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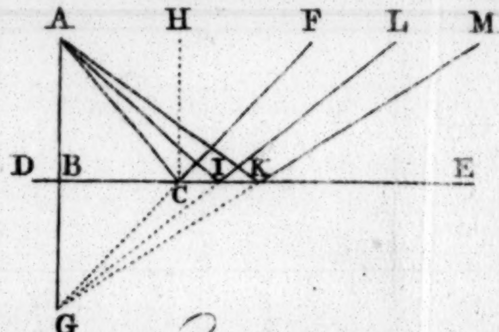

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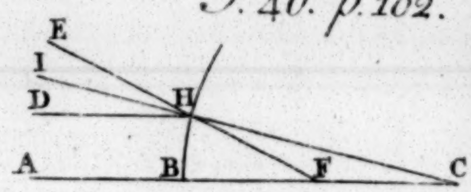

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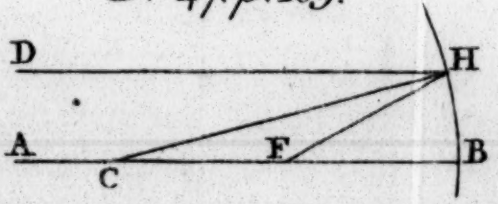
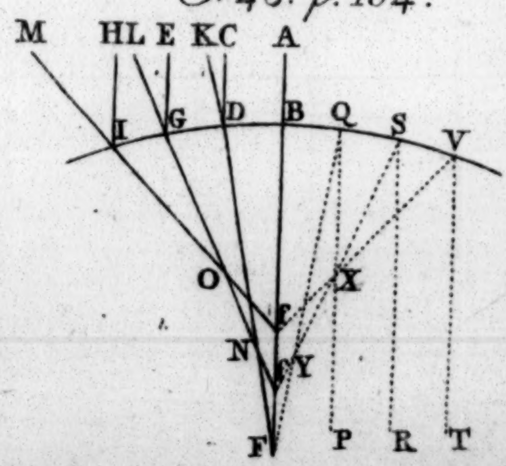


F. 48. p. 104.

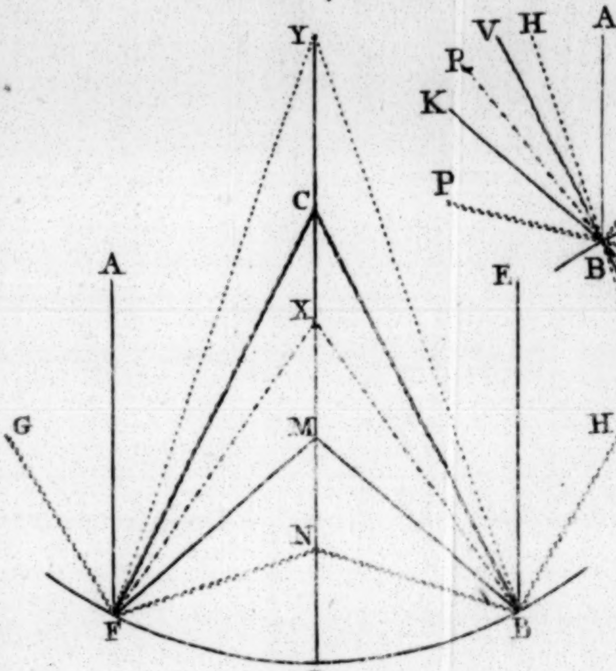


perpendicular to the Surface: then by the Observation just laid down, the Point A will appear in some Part of the Line AE produced, suppose I; that is, the oblique Rays AF and AG will proceed after Reflection as from that Point, and further, because the reflected Rays FH, GK, will have the same Degree of Inclination to each other, that their incident ones have (as was shewn in Proposition the first of the foregoing Chapter) that Point must necessarily be at the same Distance from the Surface that the Point A is; the Representation therefore of the Point A, will be at the same Distance behind the Surface, that the Point itself is before it, and directly opposite to it: consequently since the like may be shewn of the Point B, or any other, the whole Image IM will appear at the *same Distance* behind the Surface that the Object is before it, and directly *opposite* to it; and because the Lines AI, BM, which are perpendicular to the plain Surface, are for that Reason parallel to each other, it will also be of the *same Magnitude* therewith. As was to be shewn [b].

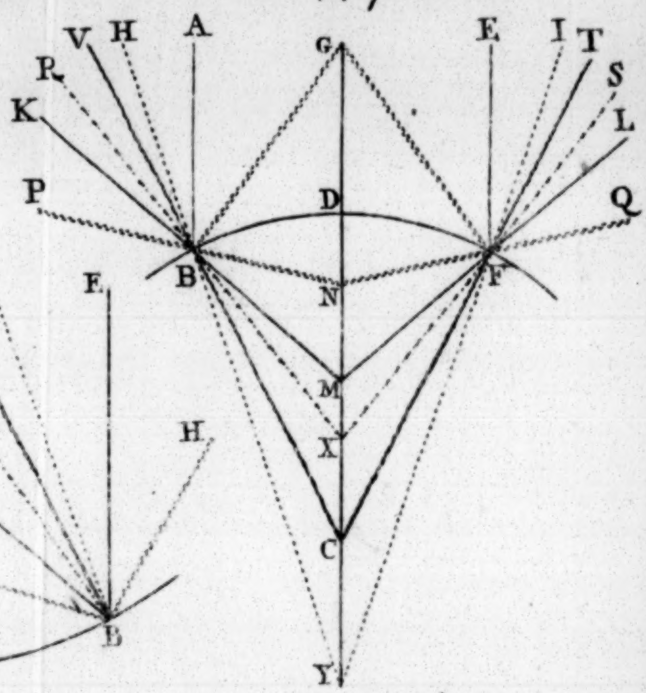
II. When an Object is seen by Reflection from a *convex* Surface, its Image appears *nearer* to the Surface, and *less* than the Object.

[b] If the Object be placed before a common Looking-glass, and viewed obliquely, three, four, or more Images of it will appear behind the Glass.

F. 43. p. 102.



F. 44. p. 110.



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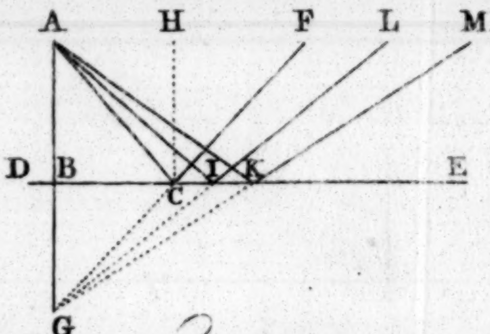

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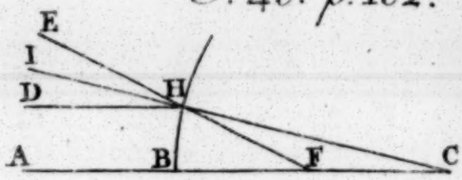

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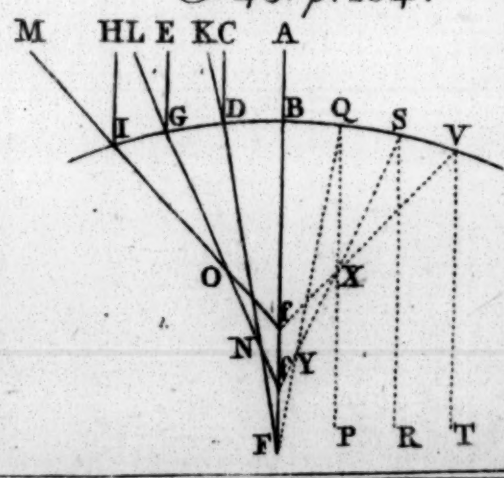

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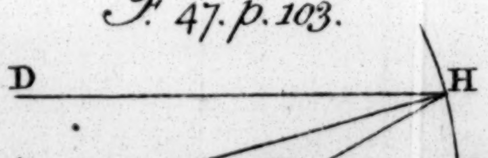

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Let AB (Fig. 54.) represent the Object, SV a reflecting Surface, whose Center of Convexity is C: and let the Rays AF, AG, be so inclined to the Surface, that after Reflection thereat, they shall enter the Eye at H: and let AE be perpendicular to the Surface: then will the oblique Rays AF, AG, proceed after Reflection as from some Point in the Line AE produced, (by the Observation laid down at the Beginning of this Chapter) suppose from I; which Point, because the reflected Rays will diverge more

To explain this, let ABCD (Fig. 53.) represent the Glass, and let EF be the *Axis* of a Pencil of Rays flowing from E, a Point in an Object situated there. The Rays of this Pencil will in Part be reflected at F, suppose into the Line FG, (see the Manner in which Light is reflected Chap. 18.) What remains will (after Refraction at F which we don't consider here) pass on to H; from whence (on Account of the Quicksilver which is spread over the second Surface of Glasses of this Kind to prevent any of the Rays from being transmitted there) they will be strongly reflected to K, where Part of them will emerge and enter an Eye at L: by this means one Representation of the said Point will be formed in the Line LK produced, suppose in M. Again, another Pencil whose *Axis* is EN, first reflected at N, then at O, and afterwards at P, will form a second Representation of the same Point at Q. And thirdly, another Pencil whose *Axis* is ER, after Reflection at the several Points R, S, H, T, V, successively, will exhibit a third Representation of the same Point at X; and so on *in infinitum*. The same being true of each Point in the Object, the whole will be represented in the like Manner; but the Representations will be faint, in Proportion to the Number of Reflections the Rays suffer and the Length of their Progress within the Glass. We may add to these another Representation of the same Object in the Line LO produced, made by such of the Rays as fall upon O, and are from thence reflected to the Eye at L.

This may be tried by placing a Candle before the Glass as at E, and viewing it obliquely, as from L.

than

than the incident ones (Prop. the sixth of the foregoing Chapter) must be nearer to the Surface than the Point A. And since the same is true also of the Rays which flow from B, or any other Point, the Representation IM will be *nearer* to the Surface than the Object; and because it is terminated by the Perpendiculars AE and BF which incline to each other, as concurring at the Center, it will also appear *less*.

III. When an Object is seen by Reflection from a *concave* Surface, the Representation of it is *various*, both with regard to its Magnitude and Situation, according as the Distance of the Object from the reflecting Surface is *greater* or *less*.

1. When the Object is nearer to the Surface, than its *Focus* of parallel Rays, the Image falls on the *opposite* Side the Surface, is more *distant* from it, and *larger* than the Object.

Thus, let AB, (Fig. 55.) be the Object, SV the reflecting Surface, F the *Focus* of parallel Rays, C its Center. Through A and B the Extremities of the Object draw the Lines, CE, CR, which will be perpendicular to the Surface, and let the Rays AR, AG, be incident upon such Points of it that they shall be reflected into an Eye at H. Now because the *Radiant* Points A and B are nearer the Surface than F the *Focus* of parallel Rays, the reflected Rays will diverge (Chap. IX. Prop. 4.) and

will

will therefore proceed as from ſome Points on the oppoſite Side the Surface; which Points, by the Obſervation laid down at the Beginning of this Chapter, will be in the Perpendiculars AE, BR, produced, ſuppoſe in I and M: but they will diverge in a leſs Degree than their incident ones (ſee the Propoſition juſt referred to) and therefore the ſaid Points will be farther from the Surface than the Points A and B. The Image therefore will be on the *oppoſite* Side the Surface with reſpect to the Object, it will be *more diſtant* than it, and conſequently, being terminated by the Perpendiculars CI and CM, it will alſo be *larger*.

2. When the Object is placed in the *Focus* of parallel Rays, the reflected Rays enter the Eye parallel (Chap. IX. Prop. 4.) in which Caſe the Image ought to appear at an *infinite* Diſtance behind the reflecting Surface; but the Repreſentation of it, for the like Reaſons that were given in the foregoing Caſe, being *large* and *diſtinct*, we judge it not much farther from the Surface than the Image [c].

3. When the Object is placed between the *Focus* of parallel Rays and the Center, the Image falls on the *oppoſite* Side the Center, is *larger* than the Object, and in an *inverted Poſition*.

Thus let AB (Fig. 56.) repreſent the Object, SV the reflecting Surface, F its *Focus* of pa-

[c] See what has been ſaid concerning the apparent Situation of Objects ſeen by parallel Rays, in Chapter VII.

rallel

rallel Rays, and C its Center. Through A and B the Extremities of the Object draw the Lines CE and CN which will be perpendicular to the Surface; and let AR, AG, be a Pencil of Rays flowing from A. These Rays proceeding from a Point beyond the *Focus* of parallel Rays will after Reflection converge towards some Point on the opposite Side the Center (Chap. IX. Prop. 4. Case 3.) which will fall upon the Perpendicular EC produced; but at a greater Distance from C than the Radiant A from which they diverged, (by the Proposition and Case just referred to.) For the same Reason, Rays flowing from B will converge to a Point in the Perpendicular NC produced, which shall be farther from C than the Point B; from whence it is evident, that the Image IM is *larger* than the Object AB, that it falls on the *contrary* Side the Center, and that their Positions are *inverted* with respect to each other.

4. If the Object be placed beyond the Center of Convexity the Image is then formed *between the Center and the* Focus *of parallel Rays,* is *less* than the Object, and its Position is *inverted.*

This Proposition is the Converse of the foregoing: for as in that Case Rays proceeding from A were reflected to I, and from B to M; so Rays flowing from I and M will be reflected to A and B; if therefore an Object

Object be ſuppoſed to be ſituated beyond the Center in IM, the Image of it will be formed in AB, *between that and the* Focus *of parallel Rays*, will be *leſs* than the Object, and *inverted*.

5. If the Middle of the Object be placed in the Center of Convexity of the reflecting Surface, the Object and its Image will be *coincident*; but the Image will be *inverted* with reſpect to the Object.

That the Place of the Image and the Object ſhould be the ſame in this Caſe needs little Explication; for the Middle of the Object being in the Center, Rays flowing from thence will fall perpendicularly upon the Surface, and therefore neceſſarily return thither again; ſo that the Middle of the Image will be *coincident* with the Middle of the Object. But that the Image ſhould be inverted is perhaps not ſo clear. To explain this, let AB (Fig. 57.) be the Object having its middle Point C in the Center of the reflecting Surface SV; through the Center and the Point R draw the Line CR which will be perpendicular to the reflecting Surface, join the Points AR and BR, and let AR repreſent a Ray flowing from A, this will be reflected into RB, for C being the middle Point between A and B the Angles ARC and CRB are equal; and a Ray from B will likewiſe be reflected to A; and therefore the Poſi-

tion of the Image will be *inverted* with reſpect to that of the Object [d].

6. If in any of the three laſt Caſes, in each of which the Image is formed on the ſame Side the reflecting Surface with the Object, the Eye be ſituated farther from the Surface than the Place where the Image falls, the Rays of each Pencil, croſſing each other in the ſeveral Points of the Image, will enter the Eye as from a real Object ſituated there; ſo that the Image will appear *pendulous* in the Air between the Eye and the reflecting Surface, and in the Poſition wherein it is formed, *viz. inverted* with reſpect to the Object, in the ſame Manner that an Image formed by *refracted* Light appears to an Eye placed beyond it; which was fully explained under the fourth Propoſition of the ſeventh Chapter, and therefore needs not be repeated here.

But as to what relates to the Appearance of the Object when the Eye is placed nearer to the Surface than the Image, that was not there fully inquired into. That Point ſhall therefore now be more ſtrictly examined under the following Caſe, which equally relates to *refracted* and *reflected* Light.

[d] In this Propoſition it is to be ſuppoſed that the Object AB is ſo ſituated with reſpect to the reflecting Surface, that the Angle ACR may be right; for otherwiſe the Angles ARC and BRC will not be equal, and Part of the Image will therefore fall upon the Object and part off.

7. If the Eye be ſituated between the reflecting Surface and the Place of the Image, the Object is then ſeen *beyond* the Surface; and the farther the Eye recedes from the Surface towards the Place of the Image, the *more confuſed*, *larger*, and *nearer* the Object appears.

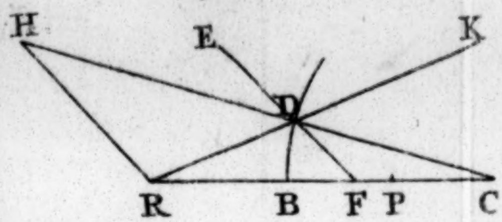
To explain this, let AB (Fig. 58.) repreſent the Object, IM its Image, one of whoſe Points, M, is formed by the Concurrence of the reflected Rays DM, EM, *&c.* which before Reflection came from B; the other, I, by the Concurrence of DI, EI, *&c.* which came from A: and let *ab* be the Pupil of an Eye ſituated between the Surface DP and the Image. This Pupil will admit the Rays Ha, Kb, which, becauſe they are tending towards I, are ſuch as came from A, and therefore the Point A will appear diffuſed over the Space RS. In like Manner the Pupil will alſo receive into it the reflected Rays Ka and Lb, which, becauſe they are tending towards M, by Suppoſition came from B; and therefore the Point B will be ſeen ſpread as it were over the Space TV, and the Object will ſeem to fill the Space RV; but the Repreſentation of it will be *confuſed*, becauſe the intermediate Points of the Object, being equally inlarged in Appearance, there will not be Room for them between the Points S and T, but they will coincide in part one with another; for Inſtance, the Appearance of that Point in the Object, whoſe Repreſentation falls upon *c* in

the Image, will fill the Space *mn*, and ſo of the reſt. Now if the ſame Pupil be removed into the Situation *ef*, the reflected Rays E*e* and G*f* will then enter the Eye, and therefore one Extremity of the Object will appear to cover the Space XY; and becauſe the Rays O*f* and L*e* will alſo enter it in their Progreſs towards M, the Point B, from whence they came, will appear to cover ZV; the Object therefore will appear *larger* and *more confuſed* than before. And when the Eye recedes quite to the Image, it ſees but one ſingle Point of the Object, and that appears diffuſed all over the reflecting Surface: for Inſtance, if the Eye recedes to the Point M, then Rays flowing from the Point B enter it upon whatever Part of the Surface they fall :and ſo for the reſt. The Object alſo appears *nearer* to the Surface, the farther the Eye recedes from it towards the Place of the Image, probably becauſe as the Appearance of the Object becomes more and more confuſed, its Place is not ſo eaſily diſtinguiſhed from that of the reflecting Surface itſelf, till at laſt when it is quite confuſed (as it is when the Eye is arrived at M) they both appear as one, the Surface aſſuming the Colour of the Object [e].

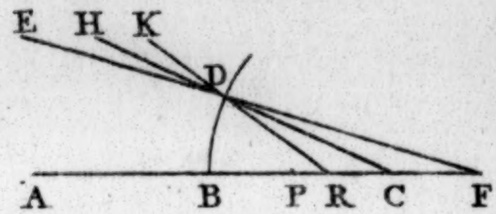
As

[e] As to the preciſe apparent Magnitude of an Object ſeen after this Manner, it is ſuch that the Angle it appears under ſhall be equal to that which the Image of the ſame Object would appear under, were we to ſuppoſe it ſeen from the ſame Place: that is, the apparent Object (for ſuch I muſt call it to diſtinguiſh

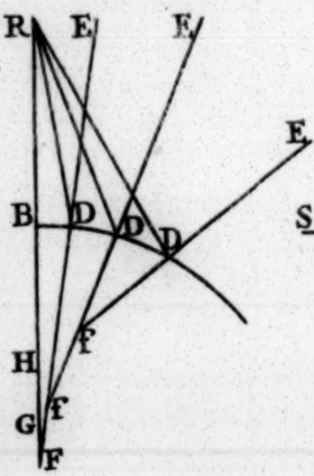
F. 49. p. 105.



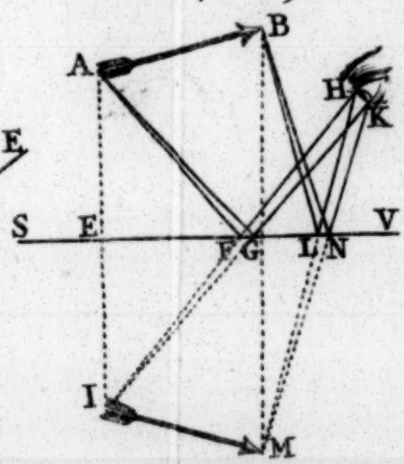
F. 50. p. 106.



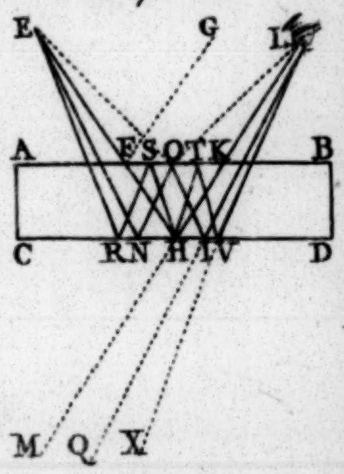
F. 51. p. 113.



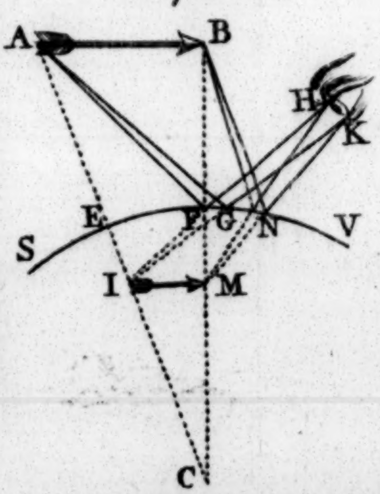
F. 52. p. 119.



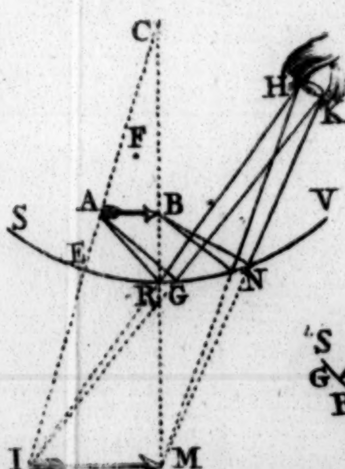
F. 53. p. 120.



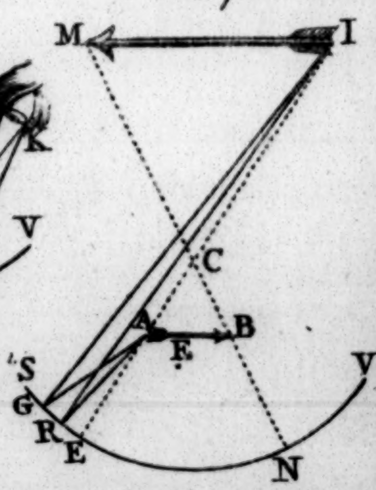
F. 54. p. 121.



F. 55. p. 122



F. 56. p. 123



As each Point in the Repreſentation of an Object made by Reflection is ſituated ſomewhere in a right Line that paſſes through its correſpondent Point in the Object, and is perpendicular to the reflecting Surface, as was ſhewn in the Beginning of this Chapter; we may from hence deduce a moſt eaſy and expeditious Method of determining both the

diſtinguiſh it from the Image of the ſame Object) and the Image ſubtend equal Angles at the Eye.

Dem. Here we muſt ſuppoſe the Pupil of the Eye to be a Point only, becauſe the Magnitude of that cauſes ſome ſmall Alteration in the apparent Magnitude of the Object; as we ſhall ſee by and by. Let then the Point *a* repreſent the Pupil, then will the extreme Rays that can enter it be H*a* and K*a*, the Object therefore will appear under the Angle H *a* K, which is equal to its vertical one M*a*I under which the Image IM would appear, were it to be ſeen from *a*. Again if the Eye be placed in *f*, the Object appears under the Angle G *f* O equal to I *f* M which the Image ſubtends at the ſame Place, and therefore the apparent Object and Image of it ſubtend equal Angles at the Eye. Q. E. D.

Now if we ſuppoſe the Pupil to have any ſenſible Magnitude, ſuch ſuppoſe that its Diameter may be *a b*, then the Object ſeen by the Eye in that Situation will appear under the Angle H *x* L, which is larger than the Angle H *a* K under which it appear'd before; becauſe the Angle at *x* is nearer than the Angle at *a*, to the Line IM, which is a Subtenſe common to them both.

From this Propoſition it follows, that, were the Eye cloſe to the Surface at K, the real and apparent Object would be ſeen under equal Angles (for the real Object appears from that Place under the ſame Angle that the Image does, as will be ſhewn at the End of this Chapter) therefore when the Eye is nearer to the Image than that Point, the Image will ſubtend a larger Angle at it than the Object does; and conſequently ſince the Image and apparent Object ſubtend equal Angles at the Eye, the apparent Object muſt neceſſarily be ſeen under a *larger* Angle than the Object itſelf, wherever the Eye be placed between the Surface and the Image.

Magnitude and Situation of the Image in all Cases whatever. Thus,

Through the Extremities of the Object AB, and the Center C, (Fig. 59, 60, or 61.) draw the Lines AC, BC, and produce them as the Case requires; these Lines will be perpendicular to the reflecting Surface, and therefore the Extremities of the Image will fall upon them. Through F the middle Point of the Object and the Center, draw the Line FC and produce it till it passes through the reflecting Surface, this will also be perpendicular to the Surface. Through G, the Point where this Line cuts the Surface, draw the Lines AG and BG and produce them this way or that, till they cross the former Perpendiculars; and where they cross, there I and M the Extremities of the Image will fall. For supposing AG to be a Ray proceeding from the Point A and falling upon G, it will be reflected to B; because FA is equal to FB, and FG is perpendicular to the reflecting Surface; and therefore the Representation of the Point A will be in BG produced as well as in AC, consequently it will fall on the Point I where they cross each other. Likewise the Ray BG will for the same Reason be reflected to A, and therefore the Representation of the Point B will be in AG produced as well as in some Part of BC, that is, in M where

where they croſs. From whence the Propoſition is clear.

If it happens that the Lines will not croſs which way ſoever they are produced, as in (Fig. 62.) then is the Object in the *Focus* of parallel Rays of that Surface, and has no Image formed in any Place whatever. For in this Caſe the Rays AH, AG, flowing from the Point A, become parallel after Reflection in the Lines HC, GB, and therefore do not flow as to or from any Point: in like Manner Rays flowing from B are reflected into the parallel Lines KB and GA; ſo that no Repreſentation can be formed by ſuch Reflection.

From hence we learn another Circumſtance relating to the Magnitude of the Image made by Reflection; *viz.* that it ſubtends the ſame Angle at the *Vertex* of the reflecting Surface that the Object does. This appears by Inſpection of the 59, 60, or 61ſt Figure, in each of which the Angle IGM, which the Image ſubtends at G the *Vertex* of the reflecting Surface, is equal to the Angle AGB, which the Object ſubtends at the ſame Place; for in the two firſt of thoſe Figures they are vertical, in the third they are the ſame. And

Farther, the Angle ICM, which the Image ſubtends at the Center, is alſo equal to the Angle ACB which the Object ſubtends at the ſame Place; for in the two firſt Figures they

they are the ſame, in the laſt they are vertical to each other.

From whence it is evident, that the Object and its Image are to each other in Diameter, either as their reſpective Diſtances from the *Vertex* of the reflecting Surface, or as their Diſtances from the Center of the ſame.

IV. As Objects are multiplied by being ſeen through tranſparent *Media*, whoſe Surfaces are properly diſpoſed, as was explained (Chap. VII. Prop. 4.) ſo they may alſo by reflecting Surfaces. Thus,

1. If two reflecting Surfaces be diſpoſed at right Angles, as the Surfaces AB, BC, (Fig. 63.) an Object at D may be ſeen by an Eye at E, after one Reflection at F, in the Line EF produced; after two Reflections, the firſt at G, the ſecond at H, in the Line EH produced; and alſo after one Reflection made at A, in the Line EA produced.

2. If the Surfaces be parallel, as AB, CD, (Fig. 64.) and the Object be placed at E and the Eye at F, the Object will appear multiplied an *infinite* Number of Times: Thus, it may be ſeen in the Line FG produced, after one Reflection at G; in the Line FH produced, after two Reflections, the firſt at I, the ſecond at H; and alſo in FP produced, after ſeveral ſucceſſive Reflections of the Ray EL, at the Points L, M, N, O, and P: and ſo on *in infinitum*. But the greater the Number of Reflections

flections are, the weaker the Repreſentation will be.

There are reflecting Surfaces made, ſome cylindrically convex, others cylindrically concave; both which Kinds are deſigned to render the Image of an Object deformed, or the contrary, by augmenting or diminiſhing it in one Dimenſion and not in another; for the former Sort are convex one way, *viz.* round their *Axes*, and plain the other, *viz.* lengthwiſe; the other are concave round their *Axes*, and plain the other way: upon which Account Surfaces of theſe Kinds neceſſarily have ſuch Effects.

When a very deformed Picture, or rather no Picture at all, but a ſeemingly irregular and accidental Poſition of Colours is placed before one of theſe Surfaces, and ſeen by Reflection from thence, a beautiful and well proportion'd Image ſhall appear therein. But to explain *Phænomena* of this Kind is not the Intent of this *Compendium*. Enough has been ſaid to explain the Principles of *Catoptrics*. Paſs we on now to the third and laſt Diviſion of *Optics*, viz. *the Doctrine of Colours*.

CHAP. XI.

Of the different Refrangibility *in the* Rays *of* Light; *of the* Colours *the* distinct Species *of them are disposed to excite*; *and of the Cause of that* Variety *of* Colours *which is observable in Bodies.*

IN treating of the Refraction of Light in the Beginning of this Part, we supposed that all Light, in passing out of one *Medium* into another of different Density, is equally refracted in the same or like Circumstances. This is the Notion the Philosophers before Sir *Isaac Newton*'s Time had of it; but that indefatigable and circumspect Author has discovered that it is not so, but that *there are different* Species *of Light*; *and that each* Species *is disposed both to suffer a different Degree of Refrangibility in passing out of one* Medium *into another, and to excite in us the* Idea *of a different Colour from the rest*; *and that Bodies appear of that Colour which arises from the Composition of the Colours the several* Species *they reflect are disposed to excite.*

There are Abundance of Experiments made by Sir *Isaac Newton* and others for the Confirmation

firmation of this Doctrine; I ſhall only ſelect the following ones, which will ſufficiently illuſtrate the Propoſition, and evince the Truth of it. And

Firſt, There are different *Species* of Light, and each *Species* is diſpoſed to ſuffer a different Degree of Refrangibility, and to excite the *Idea* of a different Colour.

To ſhew this, let a Room be darkened, and the Sun permitted to ſhine into it through a ſmall Hole in the Window-ſhutter, and be made to fall upon a Glaſs *Priſm* (by which is meant a Piece of Glaſs of a triangular Form, ſuch as is repreſented in Figure 65.) then will the Sun's Light in paſſing through this *Priſm* ſuffer different Degrees of Refraction, and by that means be parted into different Rays, which Rays being received upon a clean white Paper will exhibit the following Colours, *viz. Red, Orange, Yellow, Green, Blue, Indigo,* and a *Violet Purple.* Thus, let AB (Fig. 66.) repreſent the Window-ſhutter, C the Hole in it, DEF the Priſm, ZY a Ray of Light coming from the Sun, which paſſes through the Hole and falls upon the Priſm at Y, and if the *Priſm* were removed would go on to X, but in entring its firſt Surface EF ſhall be refracted into the Courſe YW, falling upon the ſecond in W, where in going out into the Air it ſhall be refracted again. Let the Light now, after it has paſſed the *Priſm,* be received upon a

Sheet of white Paper GHIK held at a proper Diſtance, and it will exhibit upon the Paper a Picture or Image at LM of an oblong Figure, whoſe Ends are ſemicircular and Sides ſtrait. And it ſhall be variegated with Colours after the following Manner. From the Extremity M to ſome Length ſuppoſe to the Line *no*, it ſhall be of an intenſe *Red*; from *no* to *pg* it ſhall be of an *Orange* Colour; from *pg* to *rs* it ſhall be *Yellow*; from thence to *tu* it ſhall be *Green*; from thence to *wx* *Blue*; from thence to yz *Indigo*; and from thence to the End *Violet*. And if the whole Image be divided lengthwiſe into 360 equal Parts, the *Red* ſhall take up 45 of them, the *Orange* 27, the *Yellow* 48, the *Green* 60, the *Blue* 60, the *Indigo* 40, and the *Violet* 80[f].

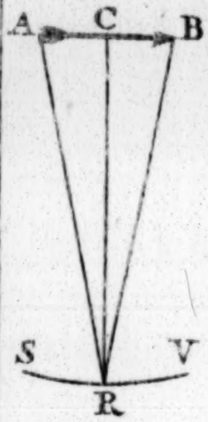
To

[f] Sir *Iſaac Newton* in his *Opitics* has ſhewn, how from the Refraction of the moſt refrangible and leaſt refrangible Rays, to find the Refraction of all the intermediate ones. His Rule is this, if the Sine of Incidence be to the Sine of Refraction in the leaſt refrangible Rays as AV to BC (Fig.67.) and to the Sine of Refraction in the moſt refrangible as AV to BD; and if CE be taken equal to CD, and then ED be ſo divided in F, G, H, I, K, L, that ED, EF, EG, EH, EI, EK, EL, EC, may be proportional to the eight Lengths of muſical Chords, which ſhall ſound the Notes in an Octave, ED being the Length of the *Key*, EF the Length of the *Tone* above that Key, EG the Length of the *leſſer Third*, EH of the *Fourth*, EI of the *Fifth*, EK of the *greater Sixth*, EL of the *Seventh*, and EC of the *Octave* above that Key; that is, If the Lines ED, EF, EG, EH, EI, EK, EL, and EC bear the ſame Proportion to each other as the Numbers

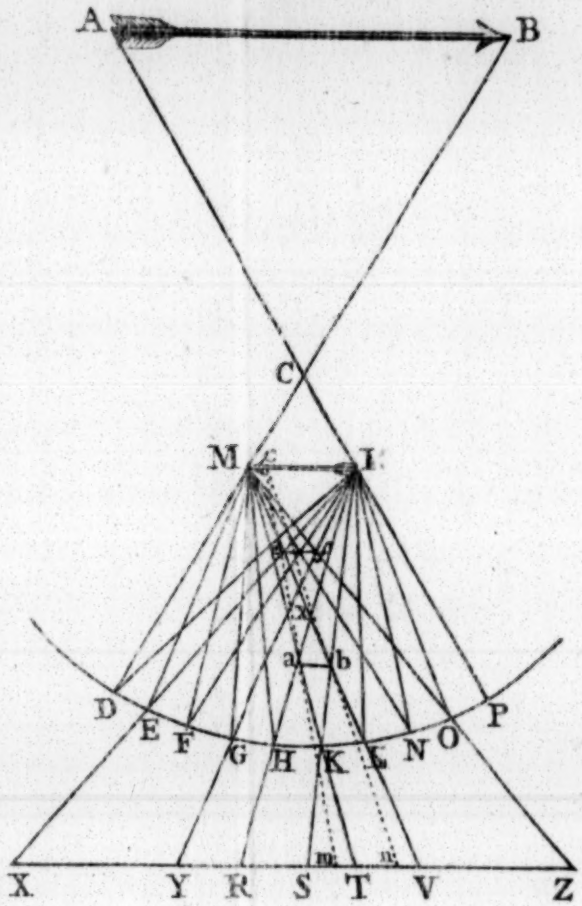
1;

Part III. Plate IX. p. 136.

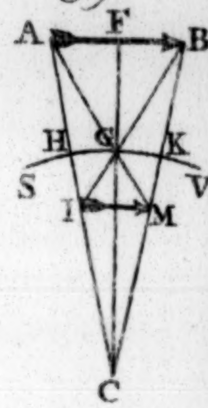
F. 57.



F. 58. p. 127.

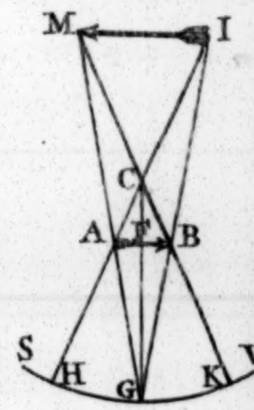


F. 59.

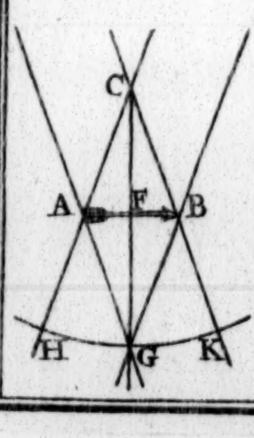


F. 60.

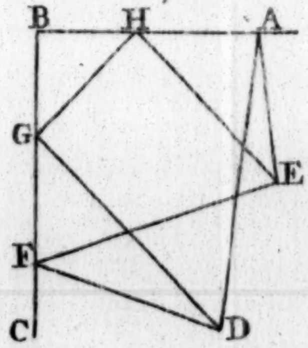
F. 61.



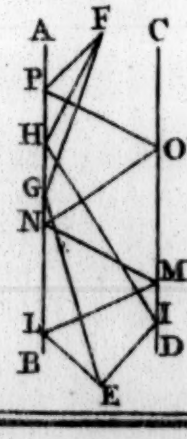
F. 62.



F. 63. p. 132.



F. 64.



F. 65.

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To render this Proof complete, we muſt now ſhew, that theſe Diſpoſitions of the Rays of Light, to produce ſome one Colour, and ſome another, which manifeſt themſelves after being refracted, are not wrought by any Action of the Priſm upon them, but are originally inherent in thoſe Rays; and that the Priſm only affords each *Species* an Occaſion of ſhewing its diſtinct Quality, by ſeparating them one from the other, which before, while they were blended together in the unfracted Light of the Sun, lay concealed.

This will be proved by the following Experiment. Things remaining as in the foregoing one,

1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, reſpectively, then ſhall BD and BF be the Limits of the Sines of Refraction of the *violet* Rays; that is, the *violet* coloured Rays ſhall not all of them have preciſely the ſame Sine of Refraction, but none of them ſhall have a greater Sine than BD nor a leſs than BF, though there be *violet* coloured Rays which anſwer to any Sine of Refraction that can be taken between theſe two. In the ſame manner BF and BG are the Limits of the Sines of Refraction of the *Indigo*; BG and BH are the Limits belonging to the *Blue*; BH and BI, the Limits pertaining to the *Green*; BI and BK, the Limits for the *Yellow*; BK and BL, the Limits for the *orange* coloured Rays; and laſtly BL and BC, thoſe of the Sines of Refraction belonging to the *Red*.

And particularly, when Light paſſes out of Glaſs into Air, if the Sine of its Angle of Incidence be 50, the Sine of the Angle of Refraction of the *Red* will be between 77 and $77\frac{1}{8}$, of the *Orange* coloured between $77\frac{1}{8}$ and $77\frac{1}{5}$, of the *Yellow* between $77\frac{1}{5}$ and $77\frac{1}{3}$, of the *Green* between $77\frac{1}{3}$, and $77\frac{1}{2}$, of the *Blue* between $77\frac{1}{2}$. and $77\frac{2}{3}$, of the *Indigo* between $77\frac{2}{3}$ and $77\frac{7}{9}$ and of the *violet* coloured Rays between $77\frac{7}{9}$ and 78.

let another Priſm, as NO (Fig. 68.) be placed either cloſe to, or at ſome Diſtance from the firſt, in a perpendicular Situation with reſpect to the former, ſo that it may refract the Rays iſſuing from the firſt ſideways. Now, if this Priſm could ſeparate the Light which falls upon it into coloured Rays, as the other did, it would divide the Image breadthwiſe into Colours, as before it was divided lengthwiſe; but no ſuch Thing is obſervable: For the Image ſhall only be thrown out of the perpendicular Situation LM into the oblique one PQ; the upper Parts, which were more refracted in the former Caſe, being more refracted in this, and therefore made to recede farther ſideways from their former Situation L, than the lower ones are from M. And farther, each Colour ſhall be uniform from Side to Side in the oblique Image, as well as in the perpendicular one.

If there be any Objection againſt the Sufficiency of this Proof, it muſt be, that the Rays, when they fall upon the ſecond Priſm, are not all in like Circumſtances, with regard to their Inclination to its Surface; I ſhall therefore, to obviate that Objection, add one more Experiment which ſeems to be peculiarly adapted to that Purpoſe. It is as follows:

Two boards AB, CD, (Fig. 69.) being erected in a darkened Room at a proper Diſtance, one of them AB, being near the Window-ſhutter EF, A Space being only left for the

Priſm

Priſm GHI to be placed between them; ſo that Part of the Rays, which enter the Hole M, may, after paſſing through the Priſm, be tranſmitted through a ſmaller Hole K made in the Board AB, and paſſing on from thence go out at another Hole L, made in the Board CD, of the ſame Size as the Hole K, and ſmall enough to tranſmit the Rays of one Colour only at a Time: Let another Priſm PQR be placed behind the Board CD to receive the Rays paſſing through the Holes K and L, and after Refraction by that Priſm, let the Rays fall upon the white Surface ST. Suppoſe, firſt, the *violet* Light to paſs through the Holes, and to be refracted by the Priſm PQR to *s*, which, if that Priſm were not there, would have paſſed on to W. If the Priſm GHI be turned about ſlowly, ſo that the incident Ray ZY may fall more obliquely upon it, while the Boards and the other Priſm remain fixed, in a little Time another Colour, ſuppoſe *Indigo*, which we may ſuppoſe before to have proceeded to *i*, will paſs through the Holes K and L, and, if the Priſm PQR were away, would proceed like the former Rays to the ſame Point W. Now the Refraction of this Priſm will not carry theſe Rays to *s* as it did the other, but to ſome Place leſs diſtant from W, as to *t*. But it is manifeſt that the Holes K and L being in the ſame Situation in each Caſe, both Sorts of Rays enter the Priſm PQR under the ſame Cir-

Circumſtances, for they are equally inclined to its Surface RP, and enter it at the ſame Point thereof; which ſhews that the one *Species* is more diverted out of its Courſe by Refraction than the other is, when the Circumſtances of Incidence are the ſame in each. Farther, if the Priſm GHI be turned about till the Rays which exhibit *Blue* paſs through the Hole L, theſe will fall upon the Surface ST below *t*, as at *u*, and therefore are ſubject to a leſs Degree of Refraction than ſuch as produce *Indigo*. And thus by proceeding it will be found that the *Green* is leſs refracted than the *Blue*, and ſo of the remaining Colours, according to the Order in which they are repreſented in an Image formed by a ſingle Priſm g.

g There are Abundance of Experiments made by the Author of this Doctrine and others for the Confirmation of it, as was obſerved above. To give them all at full Length would tire the Reader. As that if a Body be painted one half Red, and the other Blue, and then viewed through a Priſm; the apparent Place of the one half ſhall be different from that of the other: and if it be painted with a Mixture of theſe two Colours it ſhall appear confuſed and deformed. Both which evidently ſhew that the Rays, which each of theſe Colours reflect, ſuffer different Degrees of Refraction in paſſing through the Priſm. And if two Bodies be painted, the one Red and the other Blue, and the Rays which flow from them be made to paſs through a convex *Lens*, the *Focus*, made by the Concurrence of the Rays which flow from that which is painted with Blue, ſhall fall at a leſs Diſtance behind the *Lens*, than that which is made by thoſe which come from the Red one. See the Experiment made by Dr. *Deſaguliers* in a very accurate Manner (Philoſoph. Tranſ. N°. 426.) in Oppoſition to *Signr. Ritzetti*, who diſputes the Concluſiveneſs of Sir *Iſaac Newton*'s Experiments.

And

And alſo each *Species* of Rays is diſpoſed to excite in us the *Idea* of a different Colour.

This is ſufficiently clear from what has been aleady ſaid, and is farther confirmed by what follows, *viz.* That whatever Species of Rays are thrown upon any Body, they make that Body appear of their own Colour. Thus *Minium* in *red* Light appears of its own Colour; but in *yellow* Light it appears yellow; and in *green* Light it appears green; in *blue*, blue; and in *violet-purple* coloured Light it appears of a purple Colour: in like manner Verdigreaſe will put on the Appearance of that Colour in which it is placed. But each of theſe Bodies appears moſt luminous and bright when enlightned with its own Colour, and dimmeſt in ſuch as are moſt remote from that. 'Tis certain therefore each Ray is diſpoſed to excite its own Colour, which is neither to be alter'd by Refraction nor Reflection.

Thus much in Confirmation of the firſt Part of the Propoſition, *viz.* That there are different *Species* of Light, that each *Species* is diſpoſed to ſuffer a different Degree of Refrangibility, and to excite in us the *Idea* of a different Colour. We proceed now to the ſecond Part of the Propoſition, *viz.*

2. That Bodies appear of that Colour, which reſults from a Compoſition of thoſe Colours,

which the ſeveral *Species* they reflect are diſpoſed to excite.

We have juſt now ſeen that each Ray, whatever be the Colour of the Body it is reflected from, is able to excite no other *Idea* than that of its own Colour, and that coloured Bodies reflect not all the different Sorts of Rays that fall upon them in equal Plenty; but ſome Sorts, *viz.* thoſe of their own Colour, much more copiouſly than others. We will now proceed to ſhew, that the other Colours may be produced from a Mixture of thoſe ſeven, which Rays of Light when ſeparated by a Priſm are diſpoſed to exhibit. From whence it will be rational to conclude, that Bodies appear of that Colour which ariſes from the Mixture of thoſe which they reflect.

1. All the *priſmatic* Colours (*viz.* thoſe which are made by the Priſm) mixed together appear white a little inclining to Yellow, ſuch as is that of the Sun's Light.

To ſhew this, let a convex *Lens* be placed between the Priſm and the Paper which receives the Image, in order that the Rays ſeparated by it may be collected into a *Focus*; and let the *Focus* fall upon the Paper, then will the Spot where it fall appears white. And that the Whiteneſs of this focal Point is owing to the Union of thoſe Colours appears from hence, that if we remove the Paper from the

focal

focal Point, and ſuffer the Rays to croſs each other in the *Focus*, and if when they have proceeded to ſome Diſtance beyond, they be then received upon the Paper, the ſame coloured Image will be exhibited, but inverted, becauſe the Rays croſs each other in the *Focus*; an evident Proof that the Whiteneſs of the Spot was owing to nothing but the Mixture of the Rays conſtituting the ſeveral Colours of the Image. But if the Rays of any particular Colour be intercepted before they are collected in the ſaid Spot, it then appears not only of a different Colour from what it did before, but different from any of the *priſmatic* Colours taken ſeparately.

Or if the Circumference of a Wheel be painted with the *priſmatic* Colours taken in the ſame Proportion with reſpect to each other in which they are exhibited in the Image made by the Priſm, and the Wheel be turned ſwiftly about, the Circumference of that Wheel ſhall appear *white*: if they are taken in other Proportions, the Colour of the Wheel when turned about will vary accordingly. From whence this Part of the Propoſition is alſo abundantly clear.

No Compoſition of theſe Colours will produce Black: That being no Colour, but the Defect, or Abſence of all Colour whatever.

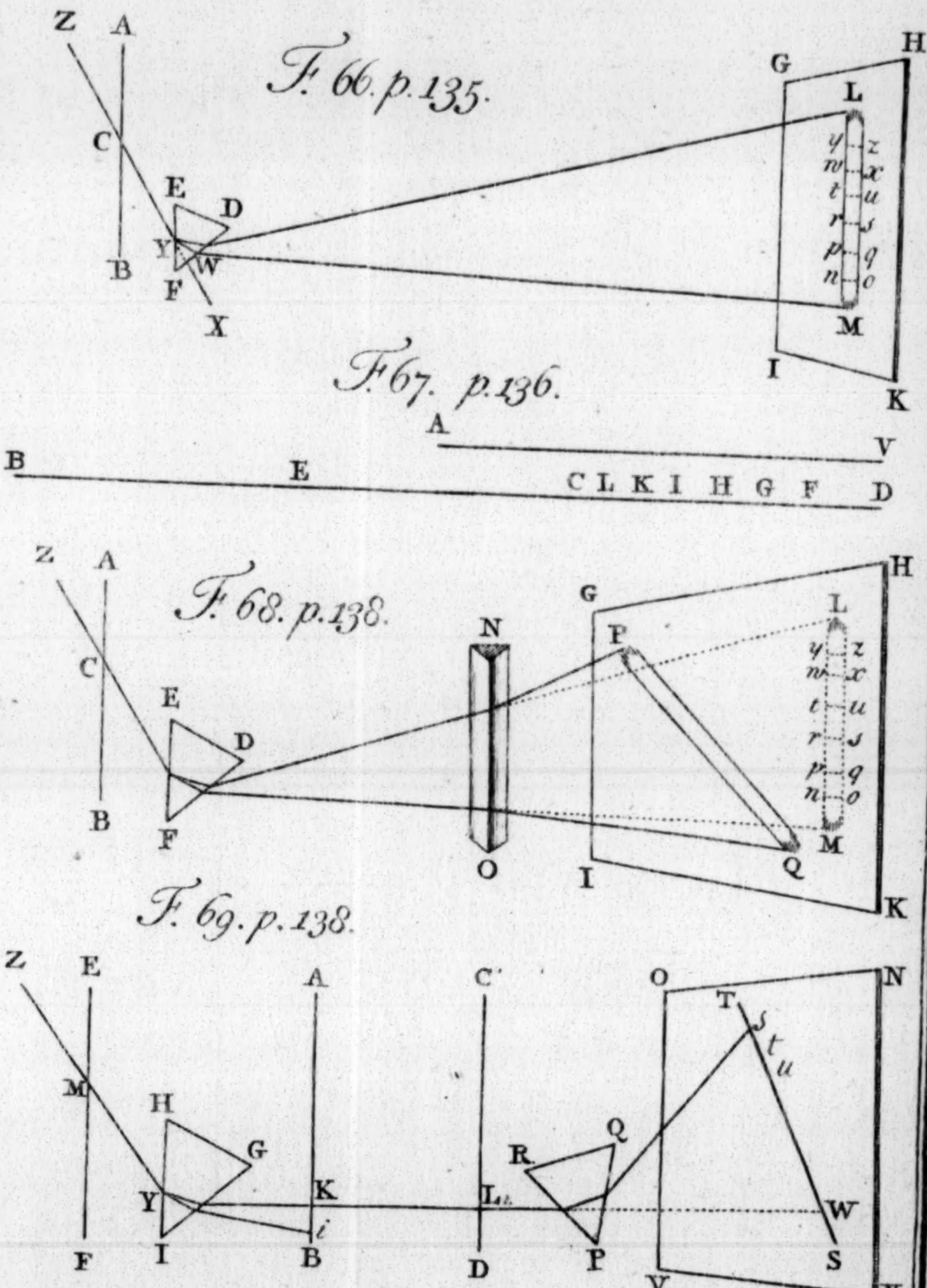
That *Species* of Light, which is diſpoſed to ſuffer a greater Degree of Refraction, requires

proportionally leſs Obliquity at the ſecond Surface of any *Medium* to occaſion a *total Reflection* of it there; ſo that it is poſſible that a Ray of Light may paſs through a *Medium* with ſuch Obliquity, that only that Part of it which is diſpoſed to exhibit a *violet* Colour ſhall be reflected at the ſecond Surface, and all the reſt tranſmitted there. This indeed is a neceſſary Conſequence of what was obſerved concerning the Reflection of Light at the ſecond Surface of any *Medium*; (Chap. 8.) *viz.* that the Reflection of a Ray is total, when the Obliquity of the incident Ray is ſuch, that the Angle of Refraction ought to be equal to, or to exceed a right one. I ſay this is a Conſequence of that, becauſe the Angle of the Refraction of the *violet* coloured Light is larger than the Angle of Refraction of any other, though their Angles of Incidence be equal. And accordingly thus it happens, as appears by the following Experiment.

Let AB (Fig. 70.) repreſent the Window-ſhutter of a darkened Room; C an Hole to let in a Ray of the Sun; DEF, GHI, two Priſms ſo applied together that the Sides EF and GI be contiguous, and the Sides DF and GH parallel: In this Situation Light will paſs through them without any Separation into Colours; for the oppoſite Sides being parallel, if the Rays are refracted one Way where they

go

Part III. Plate X. p. 144.



F. 66. p. 135.

F. 67. p. 136.

F. 68. p. 138.

F. 69. p. 138.

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go in, they will be as much refracted the contrary Way where they go out (See Page 51.) But if it be afterwards received by a third Prism KLM, it will be divided so as to form upon any white Body NOYU the usual Colours, *Violet* at *l*, *Indigo* at *m*, *Blue* at *n*, and *Red* at *r*. Now let it be supposed that the Surfaces EF and GI are not quite close together, but that the Rays, in passing from one to the other, pass through a *Medium* (*viz.* the Air) of different Density from that of the Prisms: and that the Ray ZC is not so much inclined to the second Surface of the first Prism as to cause a total Reflection of any one *Species* there; then will Part only of each *Species* be reflected and Part transmitted, agreeably to what was observed (Chapter 8.) concerning the Manner of Reflection. Let now the reflected Rays be received by a fourth Prism TXV; these, after passing through it, will paint upon a white Surface RS the Colours of the Prism, *viz.* *Red* at *s*, *Orange* at *t*, *Yellow* at *v*, and *Violet* at *z*. Let now the Prisms DEF, GHI, be slowly turned about, keeping still the same Situation with respect to each other, until the Obliquity of the Rays ZC to the Surface EF be so far increased, that there shall begin to be a total Reflection of them there. In which Case it is observable, that first of all the *violet* Light will be *totally* reflected, and will therefore disappear

at

at *l*, appearing inſtead thereof at *z*, and increaſing the *violet* Light which fell there before. And when the Rays ZC become more oblique by the Priſms being turned farther about, the *Indigo* ſhall be totally reflected, diſappearing at *m*, but falling upon *y*, and making the *violet* there more intenſe. And by turning the Priſms ſtill farther about, all the remaining Colours will be ſucceſſively removed from the Surface PQ to RS.

CHAP.

CHAP. XII.

Of the Qualifications *in Bodies, which dispose them to reflect the Rays of* different Colours.

WE are now to inquire what it is that gives Bodies this Power of reflecting, some one sort of Rays most copiously, and some another. And this is probably no other than the different Magnitude of the Particles whereof they are composed; as will appear from the following Observations.

If Water be prepared with Soap so as to render it sufficiently tenacious, and then blown up into a Bubble; it is observable, that as the Bubble grows thinner, and thinner (as it will do by reason of the Water's continually running down from the Top of it, till it breaks) different Colours will arise one after another at the Top of the Bubble, spreading themselves into Rings, and descending till they vanish at the Bottom in the same Order they arose at the Top. Thus, in an Experiment of this Kind, tried by Sir *Isaac Newton*, the Colours arose in this Order; first *Red*, then *Blue*; to which succeeded *Red* a second Time, and

Blue

Blue immediately followed; after that *Red* a third Time, ſucceeded by *Blue*; to which followed a fourth *Red*, but ſucceeded by *Green*; after this a more numerous Order of Colours, firſt *Red*, then *Yellow*, next *Green* and after that *Blue*, and at laſt *Purple*; then again, *Red*, *Yellow*, *Green*, *Blue*, *Violet* followed each other; and the laſt Order of Colours that aroſe was *Red*, *Yellow*, *White*, *Blue*; to which ſucceeded a *dark* Spot that afforded ſcarce any Light, though it was obſerved to cauſe ſome very obſcure Reflection, for the Image of the Sun or Candle might be faintly diſcerned in it; and this laſt Spot ſpread itſelf more and more, till the Bubble broke.

Now 'tis apparent that the only Reaſon, why thoſe different Colours ſucceeded each other at the Top of the Bubble in the abovementioned Manner, was becauſe its Thickneſs in that Part continually varied, till it broke. It remained therefore to examine what was the Thickneſs of the Bubble at the Top, at the Time it exhibited each particular Colour. And this was effected by the following Contrivance, *viz.* by taking the Object-Glaſs of a long Teleſcope, ſuch having but a very ſmall Degree of Convexity, and placing it upon a flat Glaſs: theſe Glaſſes by reaſon of the Convexity of the former would touch but in one Point, and the Diſtance between them, where they did not touch, would be exceedingly ſmall, but larger the farther

we

we conſider it from the Point of Contact. Now Water being put between theſe Glaſſes, the ſame Colours appeared as in the Bubble, in the Form of Circles or Rings ſurrounding the Point where the Glaſſes touched, which Point appeared black, like the Top of the Bubble when it was thinneſt. Next to this Spot lay a blue Circle, and next without that a white one, and ſo on in the ſame, but contrary Order to that in which the Colours aroſe on the Top of the Bubble.

Now the Diſtance between the Glaſſes, that is, the Thickneſs of the Body of Water between them, where it exhibited any one Colour of a particular Order, was equal to the Thickneſs of the Bubble at the Time the ſame Colour appeared upon it. For though the *Medium* the Light muſt paſs through to come at the Water is in one caſe, Glaſs, and in the other, Air; that makes no Difference in the *Species* of the Colour reflected from the Water: for Pieces of *Muſcovy* Glaſs, made thin enough to appear coloured, would have their Colours faded, but not the *Species* of them altered by being made wet with Water. But it was found that tranſparent Bodies of different Denſity would not, under the ſame Thickneſſes, exhibit the ſame Colours: for if the forementioned Glaſſes were laid upon each other without any Water between them, the Air between them would then afford the ſame Colours as the Water,

but more expanded, ſo that each Ring had a larger Diameter, though they bore all the ſame Proportion to each other; ſo that the Thickneſs of the Air proper to reflect each Colour was in the ſame Proportion larger, than the Thickneſs of the Water adapted to reflect the ſame.

Farther, all the Light which is not reflected by the thin Subſtances, whether of Air or Water contained between the Glaſſes, is tranſmitted through them; for when viewed from the other Side, they exhibit alſo coloured Rings as before, but in a contrary Order; for the middle Spot, which in the other View appears black for Want of reflected Light, now looks perfectly white; next without this Spot the Light appears tinged with a yellowiſh *Red*; where the *White* appeared before, it now ſeems *Black*; and ſo of the reſt.

It is farther obſervable, that the forementioned thin Plates, whether of Air or Water, do not appear of the ſame Colour when viewed obliquely, as when ſeen direct: for if the Rings and Colours between a convex and plain Glaſs be viewed firſt in a direct Manner, and then under different Degrees of Obliquity, the Rings will be obſerved to dilate themſelves as the Obliquity is increaſed. But a Plate of Air between the Glaſſes alters its Colour much ſooner than the Water in the Bubble which is ſurrounded with Air. For in the Water when

when viewed obliquely the ſame Colour might be ſeen at more than twelve Times the Thickneſs it appeared at under a direct View; but when the Air was viewed under ſuch an Obliquity that the Thickneſs of the Plate, where it was obſerved, was but half as much again as when it was viewed directly, a different Colour appeared.

Laſtly, the ſame Colour reflected from a denſer Subſtance reduced to a thin Plate, and ſurrounded by a rarer, will be more briſk, than the ſame Colour, when reflected from a thin Plate formed of the rarer Subſtance, and ſurrounded by the denſer, as was found by blowing Glaſs very thin, which exhibited in the open Air more vivid Colours, than the Air does between two Glaſſes.

As to the Thickneſs of the Plate of Air by which the ſeveral Colours were reflected, it was found by carefully meaſuring the Diſtances of the Rings from the Point where the Glaſſes touched, that the Diſtance between the Glaſſes where the firſt Order of Colours was reflected, was from $\frac{1}{178000}$ to $\frac{2}{178000}$ Part of an Inch; that where the ſecond, was from $\frac{3}{178000}$ to $\frac{4}{178000}$; that where the third, from $\frac{5}{178000}$ to $\frac{6}{178000}$, and ſo on in a *Series* of the odd Numbers: and that the Diſtance of the Glaſſes, where the firſt Order of Colours that was tranſmitted paſſed through, was from 0 to $\frac{1}{178000}$ Part of an

an Inch; that where the second, was from $\frac{2}{178000}$ to $\frac{3}{178000}$; that where the third, from $\frac{4}{178000}$ to $\frac{5}{178000}$, and so on in a *Series* of the even Numbers. And the Thickness of a Plate of Water, where it reflected or transmitted the same Colours, was $\frac{3}{4}$ of the Thickness of the Plate of Air.

Now we learn from Experiments made with the Microscope, that the least Parts of almost all Bodies are transparent; or the same may be experienced in the following Manner: Take a very thin Plate of the opakest Body, and the Room being darkened apply it to a small Hole in the Window-shutter, and it will sufficiently discover its Transparency. This Experiment cannot be so well performed with a white Body, because of the strong reflective Power in such; but even those, when dissolved in *Aqua Fortis* or other proper *Menstruum*, do also become transparent. Wherefore if we should suppose any Body reduced to a Thinness proper to produce any particular Colour and then broken into Fragments, in all Probability each Fragment would exhibit that Colour, and an Heap of such Fragments would constitute a Body of that Colour: so that the Cause, why some Bodies reflect one Sort of Rays most copiously, and some another, is probably no other than the different Magnitude of their constituent Particles

cles [h]. This will be farther confirmed by Particulars. The Colours in the ſame Part of a Peacock's Tail vary as the Tail changes its Poſture, with reſpect to the Eye; juſt ſo the thin Plates of Air or Water appear of a different Colour in the ſame Plate when view'd directly, from what they do when ſeen obliquely, as was obſerved above. The Colours of Silks, Cloths, and other Subſtances, which Water or Oil can intimately penetrate, become faint and dull by being wet with ſuch Fluids, and recover their Brightneſs when dry; juſt as we obſerved, that Plates of *Muſcovy* Glaſs grew faint and dim by wetting. All which Particulars, and many more that might be produced, give abundant Proof of the preſent Point [i].

[h] This Sir *Iſaac Newton* thinks a probable Ground for making Conjecture concerning the Magnitude of the conſtituent Particles of Bodies. The Green of Vegetables he takes to be of the third Order, as likewiſe the Blue of Syrup of Violets. The azure Colour of the Sky, he thinks is of the firſt Order, as alſo the moſt intenſe and luminous White; but if it is leſs ſtrong, he then conjectures it to be a Mixture of the Colours of all Orders. Of the latter Sort he takes the Colour of Linen, Paper, and ſuch like Subſtances to be; but white Metals to be of the former Sort. For producing *Black*, the Particles muſt be ſmaller than for exhibiting any of the Colours, *viz.* of a Size anſwering to the Thickneſs of the Bubble where it reflected little or no Light, and for that Reaſon appeared colourleſs.

[i] See Sir *Iſaac Newton*'s Optics, *paſſim*.

CHAP.

CHAP. XIII.

Of the Cauſe of Opacity *and* Tranſparency *in Bodies.*

WHEN two *Mediums* or tranſparent Subſtances of equal Denſity are contiguous, or as near to each other as the Glaſſes were, where the Light was wholly tranſmitted, in the Experiment made with the Object Glaſs mentioned in the foregoing Chapter, a Ray of Light will paſs from one to the other without ſuffering either Reflection or Refraction; but if they differ in Denſity, the Light will undergo both; Part of it being reflected and Part refracted. Juſt ſo it is with a Ray of Light in paſſing through the different Particles of the ſame Body. For Inſtance, if when the Ray has paſſed through any one Particle of a Body, it finds another contiguous to it, it will enter that Particle without Interruption; but if at its Emerſion out of that Particle, it enters a Pore ſufficiently large, Part of it will be tranſmitted and Part reflected. Thus will the Light every time it enters a Pore, unleſs it be an exceeding ſmall one, be in part reflected: So that nothing more ſeems neceſſary to render a Body opake, than that the Particles, of which it is compoſed, touch but in few Points, and that the Pores

Pores of it be very numerous and large; ſo that the Light, which enters it, may by numerous Refractions and Reflections be ſtifled and loſt within it. On the contrary, if the Particles of a Body touch each other in many Points, and its Pores be few and ſmall, or filled with a Subſtance of nearly an equal Denſity with the Particles of the Body, that Body will be tranſparent.

In confirmation of this, we may obſerve, that opake Bodies become tranſparent by filling their Pores with a Subſtance nearly of the ſame Denſity with that of their ſolid Parts: As when Paper is made wet with Water or Oil; when Linen Cloth is dipped in Water, oiled, or varniſhed, or the *Oculus Mundi* Stone ſteeped in Water. Beſides, as filling the Pores of an opake Body renders it tranſparent, ſo on the other hand evacuating the Pores of a Body that is tranſparent, or ſeparating the Parts of it from one another, makes it opake; as Salt or wet Paper by being dried, and Glaſs by being powder'd, loſe their Tranſparency, or Water beat up into Froth.

Beſides which Inſtances, abundance more might be brought in Confirmation of what is here laid down; but theſe are ſufficient.

But becauſe it may be Matter of Surprize, that Bodies ſhould be ſufficiently porous to tranſmit Light in that plenty we obſerve they

 do,

do, and yet be hard or ſolid; I ſhall conclude the Subject of this Chapter and of the Doctrine of Light and Colours, with ſhewing the Conſiſtency of ſuch a Suppoſition. In order to which, let us imagine a Body whoſe conſtituent Particles are of ſuch a Form that, when laid together, the Vacancies between them may be equal in Bigneſs to the Particles; how this may be done, and yet the Body be hard, is eaſy to comprehend. Now the ſolid Parts of a Body thus formed will be but half its Bulk; and if we ſuppoſe each conſtituent Particle of this Body to be formed of leſs Particles with Vacuities between them, equal to each Particle as before, the ſolid Parts of this Body will then be but a fourth Part of its Bulk; and if each of theſe leſſer Particles again be formed in the ſame Manner, the ſolid Parts of the Body ſhall be but one Eighth of its Bulk: and thus if the Compoſition be continued according to the ſame Rule, the ſolid Parts of the Body may be made to bear as ſmall a Proportion to its whole Magnitude as ſhall be deſired, notwithſtanding which, the Body, by means of the Contiguity of the Parts, ſhall be capable of being hard in any Degree. Thus, Matter bebeing infinitely diviſible, any of the leaſt Portion of it may be ſuppoſed to be wrought into a Body of any deſigned Dimenſions how great ſoever, and yet the Pores of that Body none of them

them greater than the ſmalleſt Magnitude propoſed at Pleaſure; and yet the Parts of the Body ſhall ſo touch, that the Body itſelf ſhall be hard or ſolid. Which ſhews that the whole Globe of Earth, nay, all the known Bodies of the Univerſe, for any thing that appears to us to the contrary, may be compoſed of no greater Quantity of Matter than what might be reduced into a Globe of an Inch Diameter, or into a *Nut-ſhell*.

DISSERTATION II.

Of the Cause *of* Reflection *of* Light.

THE Opinions of Philosophers relating to the Cause of this difficult *Phænomenon*, are principally four, which I shall here lay down and examine particularly; after which I shall give my own Thoughts, concerning it. And,

1. It was the Opinion of Philosophers before Sir *Isaac Newton* discovered the contrary, that Light is reflected by impinging upon the solid Parts of Bodies; but that it is not so, is clear from the following Reasons.

And first, it is not reflected at the first Surface of a Body by impinging against it.

For it is evident, that in order to the due and regular Reflection of Light, that is, that the reflected Rays should not be dispersed and scattered one from another, there ought to be no Rasures or Uneveness in the reflecting Surface large enough to bear a sensible Proportion to the Magnitude of a Ray of Light: because if the Surface abounds with such, the reflected Rays will rather be scattered like a Parcel of Pebbles thrown upon a rough Pavement, than reflected with that Regularity with which Light is observed to be from a

well

well poliſhed Surface. Now thoſe Surfaces which to our Senſes appear perfectly ſmooth and well poliſhed, are far from being ſo; for to poliſh is no other than to grind off the larger Eminences and Protuberances of the Metal with the rough and ſharp Particles of Sand, Emery, or Putty, which muſt of Neceſſity leave behind them an Infinity of Raſures and Scratches, which though inconſiderable with regard to the former Roughneſſes, and too minute to be diſcerned by us, muſt nevertheleſs bear a large Proportion to, if not vaſtly exceed the Magnitude of the Particles of Light.

Secondly, it is not reflected at the ſecond Surface, by impinging againſt any ſolid Particles.

That it is not reflected by impinging upon the ſolid Particles which conſtitute this ſecond Surface, is ſufficiently clear from the foregoing Argument; the ſecond Surfaces of Bodies being as uncapable of a perfect Poliſh as the firſt; and it is farther confirmed from hence, *viz.* that the Quantity of Light reflected differs according to the different Denſity of the *Medium* behind the Body: And that it is not reflected by impinging upon the Particles which conſtitute the Surface of the *Medium* behind it, is evident, becauſe the ſtrongeſt Reflection of all at the ſecond Surface of a Body, is when there is a *Vacuum* behind it. This therefore wants no farther Proof. (See the Man-

ner in which Light is reflected in Chapter the 8th.)

II. It has been thought by ſome[k]; that it is reflected at the firſt Surface of a Body, by a repulſive Force equally diffuſed over it, and at the ſecond, by an attractive Force.

1. If there be a repulſive Force diffuſed over the Surface of Bodies, that repels Rays of Light at all Times, then, ſince by increaſing the Obliquity of a Ray we diminiſh its perpendicular Force (which is that only, whereby it muſt make its Way through this repulſive Force) however weakly that Force may be ſuppoſed to act, Rays of Light may be made to fall with ſo great a Degree of Obliquity on the reflecting Surface, that there ſhall be a *total* Reflection of them there[l], and not one Particle of Light be able to make its Way through, which is contrary to Obſervation; the Reflection of Light at the firſt Surface of a tranſparent Body being never total in any Obliquity

[k] See *Muſchenbroek*, Element. Phyſic. Cap. 35.

[l] *Dem.* Let AB (Fig. 71.) repreſent the reflecting Surface, ABCD the repellent Power diffuſed over it, EF a Ray of Light incident upon it at the Point F, and let the Line EF by its Length expreſs the Force with which the Ray moves. This Force is reſolvable into the Forces EG and EH, or, which is the ſame thing HF and GF, which latter is the ſole Force by which the Ray endeavours to pierce through the repulſive Power. But this Force may be diminiſhed *in infinitum* by augmenting the Obliquity of the Ray EF, and therefore it may be made leſs than that of the repelling Power, in which Caſe the Ray will neceſſarily be reflected: and ſince the

Obliquity whatever. The *Hypotheſi* therefore in this Particular muſt be falſe *s*.

2. As to the Reflection at the ſecond Surface by the attractive Force of the Body; this may be conſidered in two Reſpects; firſt, when the Reflection is *total*; ſecondly, when it is *partial*.

And firſt, in Caſes where the Reflection is *total*, the Cauſe of it is undoubtedly that ſame attractive Force by which Light would be refracted in paſſing out of the ſame Body. This is manifeſt from that Analogy which is obſervable between the Reflection of Light at this ſecond Surface, and its Refraction there. For otherwiſe, what can be the Reaſon that the total Reflection ſhould begin juſt when the Obliquity of the incident Ray, at its Ar-

the ſame is demonſtrable of any Ray, (let it move with a greater or leſs Force than the Ray EF) the Obliquity of the Rays may be ſo great, that there ſhall be a total Reflection of them. Q. E. D.

m And yet from an Experiment made by Sir *Iſaac Newton*, (See his Optics Book III) it appears that there is a repulſive Force between Light and ſome Bodies. The Experiment is as follows. If over an Hole in a Window-ſhutter be fixed a thin Piece of Lead or the like, in which there is an Hole about the fortieth Part of an Inch in Diameter; and if when the Sun ſhines through that Hole, an Hair be held in its Ray at ſome Diſtance from the Hole; the Progreſs of the Rays after they have paſſed by the Hair will be as expreſſed in the 72d Figure, where A is a Section of an Hair, BC an Hole in a Window-ſhutter; BL, UF, DE, &c. Rays paſſing through it, in the Middle of which is placed the Hair A; and let RP be a Paper held at ſome Diſtance behind the Hair. Things being thus diſpoſed, the Ray BL ſhall fall upon O, UF upon Q, DE upon R, GH upon P, &c. as repreſented in the Figure.

 rival

rival at the ſecond Surface, is ſuch, that the refracted Angle ought to be a right one; or when the Ray, were it not to return in Reflection, ought to paſs on parallel to the Surface, without going from it? For in this Caſe it is evident, that it ought to be returned by this very Power, and in ſuch Manner that the Angle of Reflection ſhall be equal to the Angle of Incidence: juſt as a Stone thrown obliquely from the Earth, after it is ſo far turned out of its Courſe by the Attraction of the Earth, as to begin to move horizontally, or parallel to the Surface of the Earth, is then by the ſame Power made to return in a Curve ſimilar to that which it deſcribed in its Departure from the Earth, and ſo falls with the ſame Degree of Obliquity that it was thrown with.

But ſecondly, as to the Reflection at the ſecond Surface, when it is *partial*; an attractive Force *uniformly* ſpread over it, as the Maintainers of this *Hypotheſis* conceive it to be, can never be the Cauſe thereof. Becauſe, it is inconceivable that the ſame Force, acting in the ſame Circumſtances in every Reſpect, can ſometimes reflect the *Violet* coloured Rays and tranſmit the *Red*, and at other times reflect the *Red* and tranſmit the *Violet*.

This Argument concludes equally againſt a repulſive Force *uniformly* diffuſed over the firſt

firſt Surface of a Body, and reflecting Light there; becauſe ſome Bodies reflect the *Violet* and tranſmit the *Red*, others reflect the *Red* and tranſmit the *Violet* at their firſt Surface; which cannot poſſibly be upon this Suppoſition, the Rays of whichever of theſe Colours we ſuppoſe to be the ſtrongeſt.

III. Some, being apprehenſive of the Inſufficiency of a repulſive and attractive Force diffuſed over the Surfaces of Bodies and acting uniformly, have ſuppoſed, that by the Action of Light upon the Surfaces of Bodies the Matter of theſe Forces is put into an *undulatory* Motion, and that where the Surface of it is ſubſiding, Light is tranſmitted, and in thoſe Places where it is riſing, Light is reflected [n]. But this ſeems not to advance us one Jot farther; for in thoſe Caſes, ſuppoſe where *Red* is reflected and *Violet* tranſmitted, how comes it to paſs that the *Red* impinges only on thoſe Parts when the Waves are riſing, and the *Violet* when they are ſubſiding?

IV. The next *Hypotheſis*, that I ſhall take notice of, is that remarkable one of Sir *Iſaac Newton*'s *Fits of eaſy Reflection and Tranſmiſſion*, which I ſhall now explain and examine.

That Author, as far as I can apprehend his Meaning in this Particular, is of Opinion, that

[n] See *Muſchenbroek* Element.. Phyſic. Cap. 35.

Light,

Light, in its Paſſage from the luminous Body, is diſpoſed to be *alternately* reflected by and tranſmitted through any refracting Surface it may meet with; that theſe Diſpoſitions (which he calls *Fits of eaſy Reflection and eaſy Tranſmiſſion)* return ſucceſſively at equal Intervals: and that they are communicated to it at its firſt Emiſſion out of the luminous Body it proceeds from, probably by ſome very ſubtle and elaſtic Subſtance diffuſed through the Univerſe, and that in the following Manner. As Bodies falling into Water, or paſſing through the Air cauſe Undulations in each, ſo the Rays of Light may excite Vibrations in this elaſtic Subſtance. The Quickneſs of which Vibrations depending on the Elaſticity of the *Medium* (as the Quickneſs of the Vibrations in the Air, which propagate Sound, depend ſolely on the Elaſticity of the Air, and not upon the Quickneſs of thoſe in the ſounding Body) the Motion of the Particles of it may be quicker than that of the Rays; and therefore when a Ray, at the Inſtant it impinges upon any Surface, is in that Part of a Vibration of this elaſtic Subſtance which conſpires with its Motion, it may be eaſily tranſmitted, and when it is in that Part of a Vibration which is contrary to its Motion, it may be reflected. He farther ſuppoſes, that when Light falls upon the firſt Surface of a Body, none is reflected there, but

but all that happens to it there is, that every Ray that is not in a Fit of eaſy Tranſmiſſion is there put into one, ſo that when they come at the *other* Side (for this elaſtic Subſtance eaſily pervading the Pores of Bodies, is capable of the ſame Vibrations within the Body as without it) the Rays of one Colour ſhall be in a Fit of eaſy Tranſmiſſion, and thoſe of another in a Fit of eaſy Reflection, according to the Thickneſs of the Body, the Intervals of the Fits being different in Rays of a different Kind. This very well accounts for the different Colours of the Bubble and thin Plate of Air and Water (mention'd in Chap. XII.) as is obvious enough; and likewiſe for the Reflection of Light at the ſecond Surface of a thicker Body; for the Light reflected from thence is alſo obſerved to be coloured, and to form Rings according to the different Thickneſs of the Body, when not intermix'd and confounded with other Light, as will appear from the following Experiment. If a Piece of Glaſs be ground concave on one Side and convex on the other, both its Concavity and Convexity having one common Center, and if a Ray of Light be made to paſs through a ſmall Hole in a Piece of Paper held in that common Center, and be permitted to fall on the Glaſs; beſides thoſe Rays which are regularly reflected back to the Hole again, there will be others reflected

to the Paper, and form coloured Rings ſurrounding the Hole, not unlike thoſe occaſioned by the Reflection of Light from thin Plates. The ſame will happen if the Rays be reflected from a metalline *Speculum*, but the Light will not be *coloured*; which ſhews that the Colours ariſe from that Light which is reflected from the Back-ſide, and that in the following Manner: Beſide that Light which is regularly reflected from the farther Surface of the Glaſs, there is ſome reflected irregularly, which paſſing from the back Surface under different Obliquities, does as it were paſs through Glaſſes of different Thickneſſes, and therefore is in part reflected back again when it comes to the firſt Surface, and is in part tranſmitted through it, the tranſmitted Light, when received upon the white Paper exhibiting the Rings of Colours abovementioned [o].

As to the Light which is ſuppoſed to be reflected at the firſt Surface, his Opinion ſeems to be, that it is not *there* reflected, as I obſerved above, but that it really enters the Surface, and is reflected from the Back-ſide of the firſt *Series* of Particles that lie therein; ſo that according as theſe Particles are larger or ſmaller, the Rays of Light which at their Entrance into them

[o] This Experiment ſucceeds better, when the Back-part of the Glaſs is Quick-ſilver'd over.

(for

(for they are tranſparent, whether the Body they compoſe be ſo or not, *See Page* 154) are thereby put into Fits of eaſy Tranſmiſſion; at their Emerſion at the other Side are ſome in a Fit of eaſy Tranſmiſſion, others in a Fit of eaſy Reflection, according as the Interval of their Fits are large or ſmall. So that the Particles of a Body may be of ſuch a Size that they ſhall reflect the *Red* and tranſmit the *Violet*; or that they may reflect the *Violet* and tranſmit the *Red*; or, in general, that the ſtrongeſt and moſt forcible Rays may be tranſmitted, while the weaker are reflected; or the weaker may be tranſmitted, while the ſtronger are reflected.

Thus I have endeavour'd to clear up the Account Sir Isaac Newton has left us of his own Sentiments concerning this Matter. But after all, I cannot ſay, that I think his Solution the true one. It is too much clogged with Suppoſitions; neither is it conſonant to that Simplicity, Uniformity, and Regularity, with which Nature is every where obſerved to act. The Time will come, when the Principles of *Attraction* and *Repulſion* will be found alone ſufficient to account for this perplexing *Phænomenon*. Would any one that has a *Genius* for a Work of this Kind, and *Opportunity* to make the neceſſary Experiments, aſſume thoſe Sir *Iſaac Newton* has left, and

add others, as his Judgment ſhould direct him; he would ſoon be able to give us as eaſy and ſimple a Solution of the *Reflection of Light*, as we already have of any other *Phænomenon* whatever. p

p Perhaps it may be of Service to one that ſhall undertake this, to acquaint him of a Fact relating to this Matter, which every Philoſopher is not appriſed of, *viz.* That a Piece of Iron, when heated, aſſumes all the Colours of the Rainbow before it becomes red hot.

DISSER-

DISSERTATION III.

Of Microſcopes *and* Teleſcopes.

THAT the Teleſcope is of modern Invention is moſt certain; neither does it appear that Microſcopes or *Optic* Glaſſes of any Kind were known to the Ancients, though there are two Paſſages brought to ſhew that ſuch Glaſſes were of Uſe among them. The one is quoted by *Pancirollus* from *Plautus*, *Cedo Vitrum, neceſſe eſt* Conſpicillo *uti*; the other is taken from *Pliny*, *C. Julius Medicus, dum inungit* Specillum, *&c.* The former of theſe Quotations is a Fiction, no ſuch Paſſage being to be found in the Writings of *Plautus* [q]; and the Word *Specillum* in *Pliny* is not to be underſtood of an Optic-glaſs of any Kind, but of a Probe or other Inſtrument made uſe of by the Surgeons of that Time [r].

It is contended, that *Alexander de Spina*, a Native of *Piſa*, was the firſt that made the Uſe of Glaſſes known to the World; but our Countryman, Frier *Bacon*, who died one and twenty Years before him [ſ], was, in

q Vid. Lettere Memorabili del Abbate Michele Giuſtiani. Parte terza, Let. 16.
r See *Molineux*'s Dioptrics, Part II. Ch. 6.
ſ In the Year 1292.

all

all Probability, acquainted with them firſt, for he wrote a book of *Perſpective*, in which, he plainly ſhews that he did not only underſtand the Nature of convex and concave Glaſſes, but the Uſe of them when combined in Teleſcopes [t]; though he no where in that Treatiſe diſcovers the Manner in which they are to be put together.

The Teleſcope with the concave Eye-glaſs was firſt invented by a Mechanic of *Middleburg* in *Zeland*, called *Z. Johannides*, about the Year 1590, though *J. Lipperhoy*, another *Dutchman*, is Candidate for the ſame Diſcovery [u]. From whence, this Sort of Teleſcope is called *Tubus Batavus* [x].

Franciſcus Fontana, a *Neapolitan*, contends, that he was the firſt Contriver of the Teleſcope compoſed of two convex Glaſſes, which is now the common aſtronomical Teleſcope [y]; and *Rheita* pretends to be the firſt that rendered that Teleſcope fit for terreſtrial Uſes, by adding two Eye-glaſſes to it [z]. This Kind of Teleſcope is called *dioptrical*.

t See his *Perſpective*, Part III. and his *Epiſtola ad Pariſienſem*, Cap. 5.

u Vid. *Borellus* de vero Teleſcopii Inventore.

x This is by ſome called *Galileo*'s Teleſcope, as invented by him; but *Galileo* acknowledges, that it was upon hearing that the *Dutchman* had contrived one, that he effected his.

y See his *Obſervationes cæleſtium terreſtriumq; Rerum*.

z Ocul *Enoch* & *Eliæ*. Lib. IV. Hiſt. Acad. Reg. Lib. V. Sect. 1. Cap. 7.

The *Catadioptrical* or reflecting Teleſcope was invented by Sir *Iſaac Newton*; of which we ſhall give a particular Deſcription when we have explained the former Sort, and ſhewn the Defects of them.

Microſcopes are of two Kinds, Simple and Compound. The firſt Sort conſiſts of one Glaſs; the other of two or more.

The Simple Microſcope is no other than a convex *Lens*, through which, as we have ſhewn (Chap. VII. Prop. 3.) Objects appear magnified.

An Object ſeen through this Microſcope appears magnified nearly in that Proportion which the Diſtance, at which an Object would be ſeen diſtinctly with the naked Eye, bears to the focal Diſtance of the Microſcope.

Thus, let AB (Fig. 73.) repreſent the Microſcope, CD an Object placed at the focal Diſtance of parallel Rays, or ſomething nearer, that the Rays of the ſame Pencil may be parallel to each other, or rather diverging in a ſmall Degree, when they enter the Eye (this Circumſtance being requiſite to diſtinct Viſion:) And let the Microſcope be ſo ſmall, that all the Rays that paſs through it from the Object may enter the Pupil of the Eye EF at the ſame time, when placed cloſe to it

as in the Figure (for, unleſs it be ſo ſmall, it will ſcarce magnify ſufficiently to obtain the Name of a Microſcope.) Things being thus diſpoſed, the Angle under which this Object appears will be GIH, or CID; but this is nearly the ſame it would have appeared under, had there been no Microſcope interpoſed [a]. Notwithſtanding which, the Object is properly enough ſaid to appear magnified by this Microſcope, becauſe, without that, it could not have been ſeen diſtinctly at ſo ſmall a Diſtance from the Eye, but muſt have been ſituated eight or ten Inches from it; and therefore, ſince Objects appear under a larger Angle the nearer they are to the naked Eye, this Object appears larger, or is magnified by means of the Microſcope, in Proportion as it is ſeen diſtinctly at a leſs Diſtance with it than without it; that is, nearly as the focal Diſtance of the Microſcope is to that at which Objects are ſeen diſtinctly with the naked Eye [b].

[a] For had there been no Microſcope interpoſed, the Angle CID would have been in the Middle of the Pupil (See the Note in Page 61.) and therefore ſomething leſs, as being farther from the Object; but this is an *accidental* Circumſtance depending on the Thickneſs of the *Lens*, and its Diſtance from the Center of the Pupil, and therefore not conſidered in the Theory.

[b] An Object will alſo appear diſtinct though it be ſituated at a very ſmall Diſtance from the Eye, by being viewed thro' a ſmall Hole in a piece of Paper, the Reaſon of which was explained in Note Page 64. But then this Hole muſt be made ſo very ſmall, that, unleſs the Object be ſtrongly illuminated, it will appear very obſcurely through it.

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The Form of a Compound Microſcope is expreſſed in the 74th Figure, where AB repreſents a ſmall convex *Lens*, whoſe focal Diſtance is ſuch, that Rays flowing from the Point C may be collected in D; and EF is a larger *Lens* whoſe *Focus* of parallel Rays coincides with the Point D; and FG repreſents an Eye ſo ſituated that Rays proceeding from an Object at KL may enter the Pupil of it, after having paſſed through both Glaſſes. Things being diſpoſed in this Manner, the Object KL will appear *magnified* and alſo *diſtinct*.

For firſt, let RCS repreſent a Pencil of Rays flowing from the Point C, theſe will meet their *Axis* again in the Point D by Suppoſition, and croſſing there will enter the *Lens* EF diverging from its *Focus* of parallel Rays, and will therefore enter the Pupil of the Eye in Directions parallel to each other, and concur upon the *Retina* at Q; the Object will therefore appear *diſtinct*.

Secondly, A Pencil of Rays flowing from another Point of the Object, as L, will meet their *Axis* in M, and diverging from thence will, after being refracted by the Lens EF, become parallel with reſpect to each other; but with reſpect to the former they will converge, becauſe with regard to them, they diverged before they paſſed through the *Lens* EF from I, a Point more diſtant than its

Focus of parallel Rays. They will conſequently croſs them at ſome Diſtance from it, ſuppoſe at H, where the Pupil of an Eye being placed to receive them, the Point L will be repreſented at O. And for the like Reaſon, the Point K being repreſented at P, the Object will be ſeen under the Angle PHO or EHF, which, as will be demonſtrated in Note (*e*), is much larger than that under which it would have appeared to the naked Eye [c].

That Glaſs AB, which is ſituated next the Object, is called the *Object-glaſs*; that which is placed next the Eye, the *Eye-glaſs* [d].

The Proportion of magnifying, in a Microſcope of this Kind, is nearly in a *Ratio* compounded of the Proportion which the Diſtance of the Image from the Object-glaſs bears to its Diſtance from the Eye-glaſs; and of that which the Diſtance of the Ob-

[c] It may be remarked here, that when we view an Object through an Inſtrument of this Kind, we are then in reality looking at the Image of that Object through a ſingle Microſcope. Thus, it is MN the Image of the Object KL, formed by the Concurrence of the Rays of each Pencil in their reſpective *Foci*, which we ſee through the *Lens* or ſingle Microſcope EF; ſo that the Addition of the Glaſs AB is only that we may have an Image of the Object to look at, larger than the Object itſelf.

[d] In ſome Microſcopes there is a third Glaſs placed between the Object glaſs and the Image, and is called a *Middle-glaſs*. This is placed there only to bring the Rays to a *Focus* the ſooner, in order that the Image may fall nearer the Object-glaſs than it otherwiſe would do.

ject from the Eye bears to its Distance from the Object-glass [e].

R 2 After

[e] *Lemma*. Let EF (in Fig. 75.) be a *Lens*, whose *Focus* of parallel Rays is D; and let Rays flowing from the Point I be collected in H; I say, that if X be the middle of the *Lens*, IX will be to XH, as ID to DX.

Demonstration of the Lemma. On D the *Focus* of parallel Rays erect the Perpendicular DM; and let IE be an Incident, and EH a refracted Ray: then whereas Rays flowing from D, and passing through the *Lens*, would after Refraction be parallel to each other and to DX, a Line drawn from thence through the Middle of the *Lens*; Rays flowing from M and passing through the *Lens*, will also be refracted into Lines parallel to themselves, and consequently to MX that which passes through the Middle of the *Lens*: consequently EH is parallel to MX; the Triangles therefore IMX and IEH are similar; and therefore IM is to ME as IX to XH: but IM is to ME also as ID to DX, the Triangles IMD and IEX being also similar; IX is therefore to XH as ID to DX. Q. E. D.

Demonstration of the Proposition. To avoid the Confusion which might arise from the Multiplicity of Lines in the 74th Figure, let only CXH and LEH, *viz*. the *Axes* of the Rays which proceed from the Points C and L, be represented as in Fig. 75, and draw the Line LH, then will CHL be the Angle under which half the Object would be seen by the naked Eye at H; but EHX is the Angle under which the same Half appears when viewed through the Miscroscope. Now this Angle is to the former in a *Ratio* compounded of the Angle EHX to EIX, and of the same EIX, or (which is equal to it because vertical) CIL to CHL; *because the* Ratio *that any two Quantities bear to each other, is compounded of the* Ratio *which the first bears to any other, and of the* Ratio *which that other bears to the second*. But the first of these *Ratio*'s, *viz*. EHX to EIX, is as IX to XH, or which, as demonstrated in the *Lemma*, is the same thing as ID to DX; that is, as the Distance of the Image from the Object-glass to its Distance from the Eye-glass: and the other *Ratio*, *viz*. CIL to CHL, is as CH to CI, that is, as the Distance of the Object from the Eye to its Distance from the Object-glass. Therefore, &c. Q. E. D.

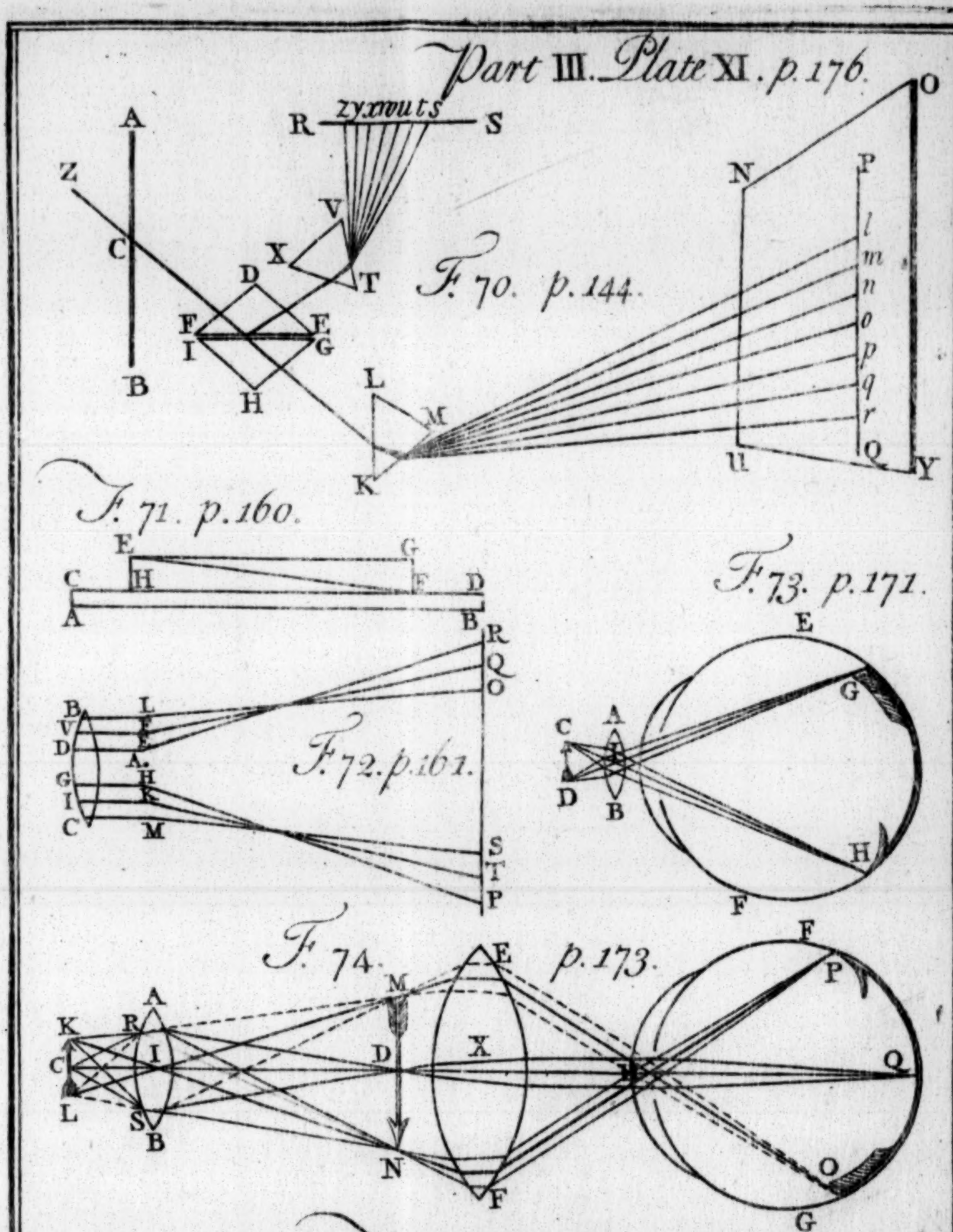
But

After what has been ſaid concerning the Structure of the Compound Microſcope, and the Manner in which the Rays paſs through it to the Eye, the Nature of the common *aſtronomical* Teleſcope will eaſily be underſtood, for it differs from the Microſcope only in that the Object is placed at ſo great a Diſtance from it, that the Rays of the ſame Pencil, flowing from thence, may be conſidered as falling parallel to one another upon the Object-glaſs; and therefore, the Image made by that Glaſs is looked upon as coincident with its *Focus* of parallel Rays.

The 76th Figure will render this very plain, in which ABC is the Object emitting the ſeveral Pencils of Rays ADF, BDF, *&c.* but ſuppoſed to be at ſo great a Diſtance from the Object-glaſs DF, that the Rays of the ſame Pencil may be conſidered as parallel to each other, they are therefore ſuppoſed to be

But it is proper to compare the Angle EHX with that under which the Object would appear to the naked Eye at a Diſtance proper for diſtinct Viſion; becauſe when a Perſon views an Object by the Help of a Microſcope, he is often obliged to place his Eye at a Diſtance from the Object, very different from that at which he would chooſe to place it, were he to look at it with the naked Eye; and then, inſtead of the Diſtance of the Object from the Eye in the foregoing Propoſition, we muſt ſubſtitute the Diſtance of diſtinct Viſion; in which Caſe it will ſtand thus: The Proportion of magnifying is nearly in a *Ratio* compounded of the Proportion which the Diſtance of the Image from the Object-glaſs bears to its Diſtance from the Eye-glaſs, and of that which the Diſtance of diſtinct Viſion bears to the Diſtance of the Object from the Object-glaſs.

Part III. Plate XI. p. 176.



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collected into their reſpective *Foci* at the Points G, H, I, ſituated at the focal Diſtance of the Object-glaſs DF. Here they form an Image, and croſſing each other proceed diverging to the Eye-glaſs KM; which being placed at its own focal Diſtance from the Points G, H, I, the Rays of each Pencil, after paſſing through that Glaſs, will become parallel among themſelves, but the Pencils themſelves will converge conſiderably with reſpect to one another, even ſo as to croſs at P, very little farther from the Glaſs KM than its *Focus*; becauſe, when they entered the Glaſs, their *Axes* were almoſt parallel, as coming through the Object-glaſs at the Point E, to whoſe Diſtance the Breadth of the Eye-glaſs KM in a long Teleſcope bears very ſmall Proportion. So that the Place of the Eye will be nearly at the focal Diſtance of the Eye-glaſs, and the Rays of each reſpective Pencil being parallel among themſelves, and their Axes croſſing each other in a larger Angle than they would do if the Object were to be ſeen by the naked Eye, as we ſhall demonſtrate in the Notes, Viſion will be *diſtinct*, and the Object will appear *magnified*.

The Power of magnifying in this Teleſcope is as the focal Length of the Object-glaſs to the focal Length of the Eye-glaſs [f].

It

[f] *Dem.* In order to prove this, we may conſider the Angle AEC as that under which the Object would be ſeen by the naked

It is evident from the Figure, that the viſible *Area*, or Space which can be ſeen at one View when we look through this Teleſcope, depends on the Breadth of the Eye-glaſs, and not of the Object-glaſs; for if the Eye-glaſs be too ſmall to receive the Rays GM, IK, the Extremities of the Object could not have been ſeen at all: a larger Breadth of the Object-glaſs conduces only to the rendering each Point of the Image more luminous by receiving a larger Pencil of Rays from each Point of the Object.

It is in this Teleſcope as was remarked of the compound Microſcope in the Notes, Page (174), that what we ſee, when we look through it, is not the Object itſelf, but only an Image of it at GI: Now that Image being inverted with reſpect to the Object, as it is, becauſe the *Axes* of the Pencils that flow from the Object croſs each other at E, Objects ſeen through a Teleſcope of this Kind neceſſarily appear *inverted.*

naked Eye; for in conſidering the Diſtance of the Object, the Length of the Teleſcope EP may be omitted, as bearing no Proportion to it. Now the Angle, under which the Object is ſeen by means of the Teleſcope, is KPM, which is to the other AEC, or its Equal KEM, as the Diſtance EL to LP, or which is the ſame Thing by *Lemma* to the foregoing Note, as EH to HL. The Angle therefore, under which an Object appears to an Eye aſſiſted by a Teleſcope of this Kind, is to that under which it would be ſeen without it, as the focal Length of the Object-glaſs to the focal Length of the Eye-glaſs.

This

This is a Circumſtance not at all regarded by Aſtronomers, but for viewing Objects upon the Earth, it is convenient that the Teleſcope ſhould repreſent them in their natural Poſture; to which Uſe the Teleſcope with three Eye-glaſſes, as repreſented Fig. 77. is peculiarly adapted, and the Progreſs of the Rays through it from the Object to the Eye is as follows:

AB is the Object ſending out the ſeveral Pencils ACD, BCD, *&c.* which, paſſing thro' the Object-glaſs CD, are collected into their reſpective *Foci* in EF, where they form an inverted Image, from hence they proceed to the firſt Eye-glaſs HI, whoſe *Focus* being at G, the Rays of each Pencil are rendered parallel among themſelves, and their *Axes*, which were nearly parallel before, are made to converge and croſs each other at K: the ſecond Eye-glaſs LM, being ſo placed that its *Focus* ſhall fall upon K, renders the *Axis* of the Pencils which diverge from thence parallel, and cauſes the Rays of each which were parallel among themſelves to meet again at its *Focus* NO on the other Side, where they form a ſecond Image inverted with reſpect to the former, but direct with reſpect to the Object. Now this Image, being ſeen by the Eye at XY through the Eye-glaſs QR, affords a direct Repreſentation of the Object, and under the ſame Angle that the firſt Image EF

EF would have appeared, had the Eye been placed at K, ſuppoſing the Eye-glaſſes to be of equal Convexity; and therefore the Object is ſeen equally magnified in this, as in the former Teleſcope, that is, as the focal Diſtance of the Object-glaſs to that of any one of the Eye-glaſſes, and appears *erect*.

If a Teleſcope exceeds 20 Feet, it is of no Uſe in viewing Objects upon the Surface of the Earth; for if it magnifies above 90 or 100 times, as thoſe of that Length uſually do, the Vapours, which continually float near the Earth in great Plenty, will be ſo magnified as to render Viſion obſcure.

The Teleſcope with the concave Eye-glaſs is conſtructed as follows:

AB (Fig. 78.) is an Object ſending forth the Pencils of Rays ADE, CDE, *&c.* which, after paſſing through the Object-glaſs, DE, tend towards FG (where we will ſuppoſe the *Focus* of it to be) in order to form an inverted Image there as before; but in their Way to it are made to paſs through the concave Glaſs HI, ſo placed that its *Focus* may fall upon S, and conſequently the Rays of the ſeveral Pencils which were converging towards thoſe reſpective focal Points F, S, G, will be rendered parallel among themſelves; but the *Axes* of thoſe Pencils croſſing each other at K, and diverging from thence, will be rendered more diverging, ſuppoſe in the

the Directions LM, NO. Now theſe Rays entering the Pupil of an Eye, will form a large and diſtinct Image P Q upon the *Retina,* which will be *inverted* with reſpect to the Object, becauſe the *Axes* of the Pencils croſs in K; and the Angle the Object will appear under will be equal to that which the Lines ML, ON, produced back through the Eye-glaſs, form at X.

'Tis evident, that the leſs the Pupil of the Eye is, the leſs is the viſible *Area* ſeen through a Teleſcope of this Kind; for a leſs Pupil would exclude ſuch Pencils as proceed from the Extremities of the Object AB, as is evident from the Figure. This is an Inconvenience that renders this Teleſcope unfit for many Uſes, and is only to be remedy'd by the Teleſcope with the convex Eye-glaſſes, where the Rays which form the extreme Parts of the Image are brought together in order to enter the Pupil of the Eye, as explained above.

It is apparent alſo, that the nearer the Eye is placed to the Eye-glaſs of this Teleſcope, the larger is the *Area* ſeen through it; for, being placed cloſe to the Glaſs, as in the Figure, it admits Rays that come from A and B, the Extremities of the Object, which it could not if it was placed farther off.

The Degree of magnifying in this Teleſcope is in the ſame Proportion with that in

the other, *viz.* as the focal Diſtance of the Object-glaſs is to the focal Diſtance of the Eye-glaſs.

For there is no other Difference but this, *viz.* that as the extreme Pencils in that Teleſcope were made to converge and form the Angle KPM (Fig. 76.) theſe are now made to diverge and form the Angle MXO (Fig. 78.) which Angles, if the concave Glaſs in one has an equal refractive Power with the convex one in the other, will be equal, and therefore each Kind will exhibit the Object magnified in the ſame Degree.

There is a Defect in all theſe Kinds of Teleſcopes, not to be remedied by any Means whatever, which was thought only to ariſe from hence, *viz.* that *ſpherical* Glaſſes do not collect Rays to one and the ſame Point, as was obſerved (Chapter III. in the Notes) but it was happily diſcover'd by Sir *Iſaac Newton*, that the Imperfection of this Sort of Teleſcope, ſo far as it ariſes from the ſpherical Form of the Glaſſes, bears almoſt no Proportion to that which is owing to the different Refrangibility of Light. This Diverſity in the Refraction of Rays is about a twenty-eighth Part of the Whole, ſo that the Object-glaſs of a Teleſcope cannot collect the Rays which flow from any one Point in the Object into a leſs Room than the circular Space whoſe

whofe Diameter is about the fifty-fixth Part of the Breadth of the Glafs [g]. Therefore, fince each Point of the Object will be reprefented in fo large a Space, and the Centers of thofe Spaces will be contiguous, becaufe the Points in the Object the Rays flow from are fo, it is evident that the Image of an Object made by fuch a Glafs muft be a moft confufed Reprefentation, though it does not appear fo when viewed through an Eye-glafs that magnifies in a moderate Degree; confequently the Degree of magnifying in the Eye-glafs muft not be too great with refpect to that of the Object-glafs, left the Confufion become fenfible.

Notwithftanding this Imperfection, a dioptrical Telefcope may be made to magnify in any given Degree, provided it be of fufficient Length; for the greater the focal Diftance of the Object-glafs is, the lefs may

g To fhew this, let AB, Fig. 79, reprefent a convex *Lens*, and let CDF be a Pencil of Rays flowing from the Point D, and let H be the Point at which the leaft refrangible Rays are collected to a *Focus*, and I, that where the moft refrangible concur; then, if IH be the twenty eighth Part of EH, IK will be a proportionable Part of EC (the Triangle HIK and HEC being fimilar:) Confequently LK will be the twenty-eighth Part of FC. But MN will be the leaft Space into which the Rays will be collected, as appears by their Progrefs reprefented in the Figure. Now MN is but about half of KL, and therefore it is but about the fifty-fixth Part of CF, fo that the Diameter of the Space, into which the Rays are collected, will be about the fifty fixth Part of the Breadth of that Part of the Glafs through which the Rays pafs. Which was to be fhewn.

be the *Proportion* which the focal Diſtance of the Eye-glaſs may bear to that of the Object-glaſs, without rendering the Image obſcure. Thus, an Object-glaſs, whoſe focal Diſtance is about four Feet, will admit of an Eye-glaſs whoſe focal Diſtance ſhall be little more than one Inch, and conſequently will magnify almoſt forty-eight times: but an Object-glaſs of forty Foot *Focus* will admit of an Eye-glaſs of only four Inch *Focus*, and will therefore magnify 120 times; and an Object-glaſs of an hundred Foot *Focus* will admit of an Eye-glaſs of little more than ſix Inch *Focus*, and will therefore magnify almoſt 200 times [h].

But

[h] The Reaſon of this Diſproportion in their ſeveral Degrees of magnifying is to be explained in the following Manner: Since the Diameters of the Spaces, into which Rays flowing from the ſeveral Points of an Object are collected, are as the Breadth of the Object glaſs, it is evident that the Degree of Confuſedneſs in the Image is as the Breadth of that Glaſs (for the Degree of Confuſedneſs will only be as the Diameters or Breadths of thoſe Spaces, and not as the Spaces themſelves.) Now the focal Length of the Eye-glaſs, that is, its Power of magnifying, muſt be as that Degree; for, if it exceeds it, it will render the Confuſedneſs ſenſible; and therefore it muſt be as the Breadth or Diameter of the Object-glaſs. The Diameter of the Object-glaſs, which is as the Square Root of its Aperture or Magnitude, muſt be as the ſquare Root of the Power of magnifying in the Teleſcope, for unleſs the Aperture itſelf be as the Power of magnifying, the Image will want Light; the ſquare Root of the Power of magnifying will be as the ſquare Root of the focal Diſtance of the Object-glaſs; and therefore the focal Diſtance of the Eye-glaſs muſt be only as the ſquare Root of that of the Object-glaſs. So that in making Uſe of an Object-glaſs of a longer *Focus*, ſuppoſe than one that is

But Teleſcopes of ſuch prodigious Length being very incommodious and unfit for Practice, the *catadioptrical* or reflecting Teleſcope as it is commonly called, invented by Sir *Iſaac Newton*, is infinitely preferable to them, for one of theſe, ſix Feet in Length, ſhall magnify as much as one of the other of an hundred. The Form of the Tube, and the Progreſs of the Rays through it, are as deſcribed in Figure the 80th, where ABCD is the Tube, BC a concave reflecting Metal, EF a plain reflecting Metal fixed to the Tube by Means of the Stem HI. MN repreſents a diſtant Object emitting Pencils of Rays from each Point, two only of which are here repreſented, and thoſe cut off before they reach the Metal, to prevent Confuſion in the Figure. Now it is evident from what has been explained above (Chap. X. Propoſit. 3. Caſe 4.) that theſe Rays, were they not intercepted in their Way, would return after Reflection at the concave Surface BC, and form an inverted Image at OP, ſuppoſing

is given, you are not obliged to apply an Eye-glaſs of a proportionably longer *Focus* than what would ſuit the given Object glaſs, but ſuch a one only whoſe focal Diſtance ſhall be to the focal Diſtance of that which will ſuit the given Object-glaſs, as the ſquare Root of the focal Length of the Object-glaſs, you make Uſe of, is to the ſquare Root of the focal Length of the given one. And this is the Reaſon that longer Teleſcopes are capable of magnifying in a greater Degree than ſhorter ones, without rendering the Object confuſed or coloured.

that

that to be the Place of the *Focus* of reflected Rays. But in this Caſe the reflected Rays are intercepted in their Return to that Place by the plain Metal, and are thereby thrown ſide-ways, and, inſtead of forming the Image OP, are made to form the Image QR; which, becauſe the Rays have as yet ſuffered no Refraction, is not liable to the Imperfection which ariſes from the different Refrangibility of the Rays of Light, nor to any other except what may ariſe from an imperfect Poliſh, or the Want of the Form of one of the *conic* Sections in the Reflector BC; and therefore may be viewed by an Eye at T with a very ſmall *Lens* or Eye-glaſs KL, without appearing either coloured or confuſed.

It being inconvenient to find the Object with a Teleſcope of this Form, a ſmall dioptrical Teleſcope with two Hairs or Wires run through the Tube in the common *Focus* of the two Glaſſes, and croſſing each other at right Angles, is generally fixed upon it in ſuch a Manner that the *Axis* of one Teleſcope ſhall be parallel to that of the other, ſo that when the Object appears in one at the Interſection of the Hairs, the other may be duly poſited for viewing the ſame Object through its Side.

But this Method of finding an Object is very incommodious for viewing terreſtrial Objects,

Objects, and therefore the fame Kind of Telefcope has been contrived and effected in the follwing Manner:

ABCDEFGH (Fig. 81.) is the Tube, BG the concave reflecting Metal, with an Hole in it at IK. LM is another reflecting Concave fix'd to the Tube by means of the Stem NO, the common *Focus* of the two Metals being at P. Things being thus difpofed, let QR reprefent an Object emitting feveral Pencils of Rays, two of which are reprefented in the Figure. Thefe, after Reflection, will form the two Extremities of the inverted Image ST (as explained Chap. X. Prop. 3. Cafe 4.) where the feveral Rays of the fame Pencil crofs each other, and, being afterwards reflected by the concave Surface LM, become parallel among themfelves, but the Pencils themfelves are made to converge, and, croffing each other at V, pafs through the *Lens* CF, which having its focal Diftance about V, makes the Pencils parallel, and at the fame Time renders the Rays of each Pencil converging, fo as to form an erect Image WX, which is feen by the Eye at Y through another *Lens* at DE.

This Kind of Telefcope is called the *Gregorian*, as being attempted by *J. Gregory*, though in vain. (See his *Optica promota*, Propofit. 59.) It is now grown common, and is excellently well adapted for the

 viewing

viewing terreſtrial Objects, becauſe a Tube of this Kind, of two or three Feet in Length, will magnify ſufficiently for that Purpoſe.

If the Reader would ſee a particular Hiſtory of the Invention of the ſeveral Sorts of reflecting Teleſcopes, he may conſult the *Appendix* to the laſt Edition of *Gregory*'s *Optics*, where he will find a full Account of it, together with the Letters that paſſed between the Inventors themſelves upon that Occaſion. And for ſuch Authors as have explained the Nature of Microſcopes and Teleſcopes in general, conſult Mr. *Johnſon*'s *Quæſtiones Philoſoph.* Q. 34 & 35.

DIS-

DISSERTATION IV.

Of the Rainbow.

BEfore the different Refrangibility of Light was discovered, all Attempts to account for the Appearance of the Rainbow proved ineffectual; for it is no other than the Diversity of Refrangibility to which that *Phænomenon* is to be ascribed: as will appear from the following Explication of it; in which, because it is a *Phænomenon* not easily apprehended by Beginners, I hope to be excused, if I am more than ordinarily particular. To begin then:

The Rainbow is never seen, but when the Sun shines upon Drops of Rain falling on that Side of the Spectator which is opposite to the Sun.

To illustrate this, let A (Fig. 82.) represent the Eye of a Spectator, and let B, C, D, E, be a Series of Drops of Rain falling from a Cloud, on which let the Sun be supposed to shine from the Parts about S, &c. then will there be exhibited the Appearance of a Rainbow in the Cloud; and it will be formed as follows. Let SB, SC, SD, &c. represent the Sun's Rays, which (because of the Sun's great Distance) we will suppose

 parallel;

parallel; and let the Ray SC fall upon the Drop C at the Point C: then will ſo much of it as enters the Drop be refracted towards the Perpendicular, and proceed on, ſuppoſe to F, where Part of it will be tranſmitted, and Part reflected, ſuppoſe to G[i]: Of that which is reflected to G, ſome will be there reflected and ſome tranſmitted; that which is tranſmitted will, on account of the Diverſity of Refrangibility to which Light is ſubject[k], be ſeparated by Refraction, and made to exhibit the ſeveral *priſmatic* Colours, *viz.* *Red*, *Orange*, *Yellow*, *&c.* And if the *Red* Light proceeds from the Drop in the Line GR, the *Orange*, ſuffering a greater Degree of Refraction, will proceed in one ſituated above this, ſuppoſe in GO, and the *Yellow* in GY, *&c.* and the *Violet* in GV; therefore, to an Eye placed any where in the Line GR, the Drop C will exhibit a *Red* Colour, that is, the Cloud will appear *Red* in that Place. To an Eye placed any where in the Line GO, the ſame Drop would exhibit the *Idea* of *Orange* Colour, and ſo on through all the Colours of the *Priſm*.

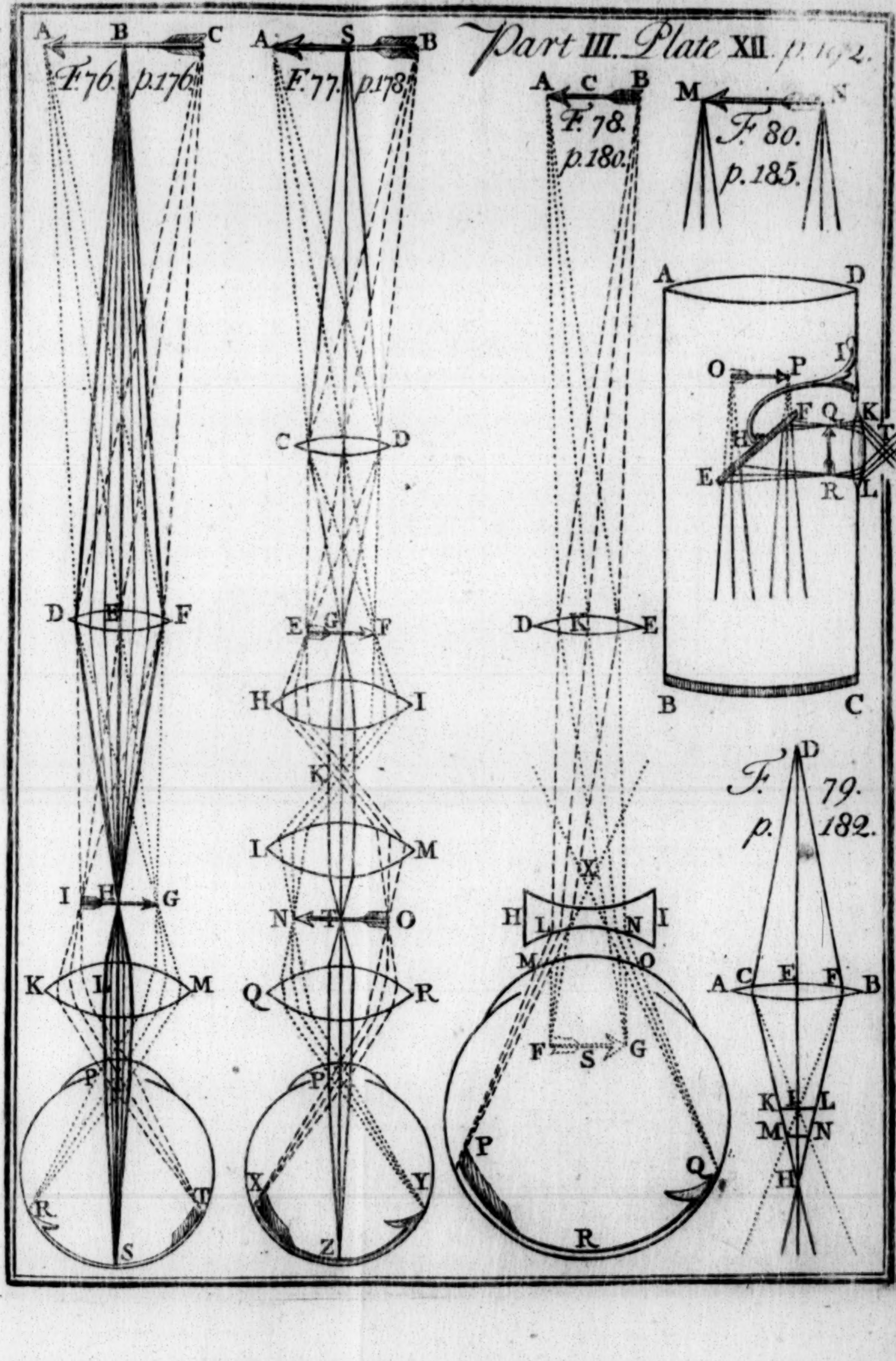
Now, let us conſider the Paſſage of a Ray of Light through another Drop at a certain Diſtance below this, *viz.* the Drop D, on

i See the Manner in which Light is reflected, Chap. VIII.
k See Chap. XI.

which

which let SD be the incident Ray. This Ray, after having been refracted and reflected as the former was in the other Drop, will emerge ſeparated into the Rays HR, HO, HY, *&c.* of which, if HR exhibits the *Red*, HO will paint the *Orange*, HY the *Yellow*, *&c.* and HV the *Violet* Colour; and the Ray incident upon this Drop being parallel to that which was incident upon the former, the Rays, exhibiting the ſeveral Colours ſeparated by this Drop, will be reſpectively parallel to the Rays exhibiting the correſpondent Colours ſeparated by means of the other Drop; that is to ſay, the Ray HR, which exhibits *Red* in this Drop, will be parallel to the Ray GR, which exhibits the ſame Colour in the other Drop; and ſo of the other correſponding Colours. Conſequently the Rays HO, HY, *&c.* which exhibit *Orange*, *Yellow*, *&c.* in this Drop, will all converge towards GR, which exhibits *Red* in the other: and therefore each of theſe would croſs that, if produced far enough. Let then the Ray HV in the Figure before us, which exhibits *Violet* in this Drop, be produced till it croſſes that which exhibits *Red* in the other produced alſo, ſuppoſe at the Point A where the Eye of the Spectator is placed. To this Eye therefore, upon this Suppoſition, will be repreſented in the Cloud at the ſame Time two of the *priſmatic* Colours, *viz. Red* and *Violet*, the

Red above at G, and the *Violet* below at H. But if we ſuppoſe the Eye placed farther back, where a Ray, that exhibits another Colour in the Drop D, would croſs the Ray GR, or which comes to the ſame Thing, if we ſuppoſe the Drop D ſo much nearer to the Drop C, that that Ray may enter the Eye along with the other at A; then would the Colour of that Ray be exhibited along with the *Red*. For Inſtance, if the Drop D be placed ſo much higher, that the Ray HO which exhibits *Orange*, may croſs the Ray GR at A, then to the Eye will be exhibited the Colours of *Red* and *Orange*; and if there be a third Drop below this, ſo placed that the *Yellow* proceeding from it ſhall enter the Eye alſo at the ſame Time, then will three of the *priſmatic* Colours appear to that Eye, and ſo on for the other Colours, till the Situation of the Drop be ſuppoſed as low as where the Drop D is, and then the *Violet* and moſt refrangible Light is tranſmitted to the Eye; but from Drops below this, no Colour is tranſmitted thither, all the Rays which iſſue from a lower Drop, as E, paſſing below the Eye. And as thoſe Rays, which paſs through the lower Drops, are too low for the Eye at A, ſo thoſe which come from the higher ones, as B, are too high, as appears by Inſpection of the Figure; ſo that there is nothing but total Darkneſs both


Part III. Plate XII. p. 172.
F. 76. p. 176
F. 77. p. 178
F. 78. p. 180.
F. 80. p. 185.
F. 79. p. 182.

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above and below. Now ſince the Colours of the Rainbow are the ſame with thoſe of the *Priſm*, it is evident, from what has been ſaid, that between the Points G and H, all the Colours of that beautiful *Phænomenon* will appear, provided a ſufficient Number of Drops be ſupplied from the Cloud, to prevent any *Hiatus* or Deficiency in the Light reflected by them.

But we have hitherto *tacitly* ſuppoſed, that the Rays SB, SC, SD, *&c.* were all incident with the ſame Degree of Obliquity upon the Surface of each Drop (that is, that they entered at the ſame Diſtance from that Point in the Surface of each Drop which is neareſt the Sun) and that that Obliquity was certain and determinate: For Rays which enter the Drops with other Obliquities conduce nothing towards exhibiting the Colours of the Rainbow, and are therefore to be looked upon as *ineffectual* and *inſignificant*. The Truth of which we ſhall now proceed to ſhew. After this, we ſhall be enabled to explain the remaining Particulars relating to the *Bow*.

Let then SA, SB, SC, *&c.* (Fig. 83.) repreſent the Sun's Rays falling upon the Drop XY, the firſt perpendicularly to the Surface of it, the other with different Degrees of Obliquity, according to their different Diſtances from the firſt; and let the Point at which the two firſt, *viz.* SA and SB, would meet by means of that Refraction which the oblique one SB

ſuffers

ſuffers in paſſing through the firſt Surface of the Drop, be H. Now it was remarked (Chap. III. Obſervat. 2. in the Notes) that when parallel Rays fall upon a convex Surface, the nearer any one of the oblique ones is to that which enters the Surface perpendicularly, the greater ſhall be the Diſtance at which it will meet the perpendicular one after Refraction at that Surface; that is, in the preſent Caſe, that the oblique incident Ray SB ſhall after Refraction at B, (ſuppoſing it to paſs through the ſecond Surface of the Drop without Refraction) meet the perpendicular Ray SA produced at a greater Diſtance than SC ſhall; and SC ſhall meet it at a greater than SD ſhall; SD at a greater than SE, *&c.* H then being ſuppoſed to be the Point where SB meets SA, let I be that where SC, K that where SD, L that where SE, M that where SF would meet it, *&c.* From whence we may obſerve,

That the farther we take the Rays from SA, the *nearer* are the Points which the refracted Rays fall upon the ſecond Surface of the Drop ſituated at X, till we come to the Ray SD; after which, the farther we take them from SA, the *farther* the Points they fall upon are from X. For Inſtance, the Ray SB falls upon N; SC upon O; SD upon P; but SE does not fall beyond P, but upon O, and SF upon N, *&c.* So that upon every Point below P there are two Rays incident; and the one of them

them is ſuch as enters the Drop on one Side of the Ray SD, and the other on the other; and the farther from it the one is on the one Side, the farther the other is on the other; and alſo the farther they are from it, the farther the Point they meet at is from P. Thus SC and SE meet at O, the Rays SB and SF at N, *&c.* The Uſe of which Obſervation we ſhall ſee by and by. But let it be remembered, that I would be underſtood here and in what follows of the Rays of one particular Colour only.

Now it is remarkable, that when two Rays fall upon a Drop, and at their Entrance are ſo refracted, as to meet in a Point at the other Surface, and are from thence reflected to ſome other Part of the Surface, and there paſs out of it; they ſhall after ſuch Emergency have the ſame Inclination towards each other, that they had before they entered the Drop. To explain this, let AB, CD, (Fig. 84.) repreſent two Rays incident upon the Drop BEF, and let them, after Refraction at B and D, meet at the Point E, from whence being reflected, let them paſs out at F and G, and be refracted into the Lines FH and GI; then whatever Inclination the incident Rays AB, CD, have to each other, the emerging Rays FH and GI will have the ſame. Becauſe the Angles of Incidence and Reflection at E being equal, the Rays EF and EG will have the ſame Inclination to each other, and to the Surface at F and G, that the Rays EB and

ED

ED have to each other, and to the Surface at B and D. For if we conceive all theſe Rays to flow from the Point E as a Radiant, and BA, DC, to be the refracted ones of the incident ones EB, ED, as FH and GI are of EF and EG, it is evident, that under theſe Circumſtances, the Rays BA and CA will have the ſame Inclination to each other, that FG and HI have; but the Degree of Refraction is the ſame, whether EB and ED, or AB and CD, be the incident Rays; becauſe the refractive Power of the Drop is the ſame, whether the Rays paſs one Way, or the other. The Propoſition therefore is true.

From hence it follows, that the parallel Rays, SB, SF, (Fig. 83.) which after Refraction meet at the ſame Point N, will if they are from thence reflected, ſuppoſe in the Lines NQ, NR, become parallel to each other, after their Emergency, ſuppoſe in the Lines QT, RV: But their intermediate ones SC, SD, and SE, which fall upon quite different Points at the ſecond Surface of the Drop, and are from thence reflected, will not do ſo, but will go out in Directions oblique to one onother and to them; and will therefore paſs on, not only a different Way from them, but from one another: So that the Rays QT and RV will be left to themſelves, being deprived of their *intermediate* ones, by which Means they are rendered, as to all Intents and Purpoſes of Viſion, entirely uſeleſs and *inſignificant*.

Again,

Again, the Rays SC and SE, which meet at the Point O, will alfo be parallel among themfelves after their Emergency from the Drop, but their *intermediate* ones will pafs off another Way, though not fo obliquely, with refpect to them and to one another, as thofe in the foregoing Cafe; becaufe the feveral Points they fall upon at the fecond Surface of the Drop, being fituated between O and P, are nearer to each other, than the Points the *intermediate* Rays fell upon, in the former Cafe, were to N.

But fuch as are incident very near SD on each Side of it, will with their intermediate ones all fall upon, or at an infenfible Diftance from the Point P; fo that thefe, after their Emergency, will all pafs on parallel, or very nearly fo, to each other; and therefore when they enter the Eye of a Spectator, though he be at a confiderable Diftance, will affect him fenfibly enough to excite the *Idea* of their own Colour (for as was obferved above, I fpeak now only of Rays of one Colour) which the other Rays confidered in the forementioned Cafes, for Want of their *intermediate* ones, were too weak to do, however near the Situation of the Eye might be. Thefe therefore are the only Rays that exhibit the Colours of the *Bow*, and are hence called in Contradiftinction to fuch as enter at other Points of the Drops, that is, with other Obliquities, *effectual* or *fignificant*.

It was proper therefore, in the Explication above, to ſuppoſe none to enter the Drop, but theſe. As to the Degree of Obliquity with which Rays muſt fall upon the Drops to become effectual, the Method of finding that will be ſhewn in the next Note.

Since then the effectual Rays enter each Drop with the ſame Degree of Obliquity (I ſtill mean ſuch Drops as exhibit the ſame Colour) the emerging Rays muſt neceſſarily make the ſame Angle with the incident ones in every Drop. The Magnitude of which ſhall be determined in the Note below [a]. That is, the Angle which the Ray SC (Fig. 82.) makes with the emerging Ray GR which exhibits *Red*, will be the ſame in all the Drops that exhibit that Colour, and ſo of the reſt. Let then the Line

[a] We are here to determine the Angle, which an incident *efficacious* Ray of any Colour makes with the emerging Ray of the ſame Colour. In order to this, let AB, CD, (Fig. 84.) be two Rays incident upon the Drop BG, and let them be refracted to E, and after Reflection there, and a ſecond Refraction at F and G, let them emerge in the Lines FH and GI, making with the incident ones the Angle AKI, which is the Angle to be determined.

Let us call the *Ratio*, which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, I to R. Then from the Center L to the Lines BK, BE, and DE, draw the Lines LM, LN, and LO, reſpectively perpendicular, and with the *Radius* LO deſcribe the Arch OP, and draw the Line LB, and produce it to Q. Then will ABQ, or its Equal LBM, be the Angle of Incidence of the Ray AB, and LM its Sine: LBN will be the Angle of Refraction, and LN its Sine: Likewiſe LR will be the Sine of the Angle of Incidence of the other Ray CD, and LO

Line AI be ſuppoſed to be drawn from the Sun through the Eye of the Spectator. This Line, becauſe of the Sun's immenſe Diſtance, will be parallel to the Sun's Rays SB, SC, SD; and therefore the Angle GAI which this Line makes with any one of the emerging Rays, for Inſtance GR, will be the ſame that the incident one makes with it. Let us then imagine the Line AI fixed, and the Line AG to revolve round it, always making the ſame Angle with it; then will the Line AG deſcribe the Surface of a Cone whoſe *Apex* will be at A, and its *Axis* AI, and the Surface of this Cone will in all Parts of it make the ſame Angle with the Sun's Rays, becauſe they are parallel to

LO the Sine of its Angle of Refraction. We ſhall therefore have for the firſt Step of the following Proceſs

this Proportion; *viz.*	1	LM : LN : : I : R
And for the ſecond	2	LR : LO : : I : R
But by Conſtruction	3	LO=LP
Therefore from the 2d and 3d Steps	4	LR : LP : : I : R
Now if we ſubſtract the two firſt Members of the fourth Step from the two firſt of the firſt Step reſpectively, by which Means the Proportion between the Terms will not be deſtroyed, we ſhall have	5	LM—LR : LN—LP : : I : R
But by the Figure	6	LM—LR=MR
And ——— ——— ———	7	LN—LP=NP
Therefore from the fifth, ſixth, and ſeventh Steps, we have	8	MR : NP : : I : R

one another, and to the Axis of the Cone; therefore Drops of Rain, whatever Part of this Surface they paſs through in falling, will, in the Inſtant of Time that they paſs through it, ſend a *red* Ray towards the Eye of the Spectator; for it is not neceſſary that the Drops ſhould be all at the ſame Diſtance from the Eye. In like manner Drops of Water, paſſing through the Surface of a leſſer Cone made by the Revolution of the Line HV about the Line AI, will exhibit *Violet*; and ſo for the intermediate Colours. So that the Rainbow, were we to ſee it entire, would be a compleat Circle having its Center in the Line AI, and conſequently directly oppoſite to the Sun with reſpect to the Spectator's Eye.

Parallel to BE draw the Line DX, and on the Lines DR and DX let fall the Perpendiculars BT and BX, from the Point B. Then, becauſe we ſuppoſe the Rays AB and CD *efficacious* ones, and therefore infinitely near one another, the little Triangles BTD, BXD, and NOP may be conſidered as right-lined ones, and the latter, *viz.* NOP may be alſo looked upon as right-angled at P. Upon this Suppoſition, the Triangles BTD and BLM will be ſimilar, *for they are right-angled at T and M; and the Angles DBT and MBL want each the ſame Angle TBL to make either of them right, they are therefore alſo equal.* For the like Reaſon the Triangles DBX and BNL are ſimilar, *being right-angled at X and N, and wanting equally the Angle XBL to make their Angles at B right ones.* But to the Triangle BDX the Triangle NPO is alſo ſimilar, *for they are right-angled at P and X, they have their Sides NP and BX parallel, as being each perpendicular to the ſame Line BE; and their Sides NO and BD are alſo parallel, becauſe the Points N and O, where the Perpendiculars LN and LO fall, muſt be in the Middle of the Lines BE and DE.* Farther, ſince the Lines BE and DE are coincident at E, and the Points N and O are in the Middle of each, BD is double of NO; and conſequently,

Eye. The Reaſon that it does not appear ſuch, is becauſe the Sun when the Rainbow is ſeen, is *in* or *above* the Horizon, and therefore the Center of the Bow being oppoſite to it, is *in* or *below* it, on the other Side the Spectator.

Since the Angle made by the Line SC with GR, or which is the ſame thing, GR with AI is 42 Degrees and two Minutes, as determined in the Note, it's evident that when the Sun is in the Horizon, the higheſt Point of the *Bow* is 42 Degrees and two Minutes above the Horizon,

quently, the Triangles NOP and BDX being ſimilar, BX is double of NP. From all which we have the following Steps, *viz.*

viz.	9	BL : BM :: BD : BT
And	10	BL : BN :: BD : BX
By comparing theſe two together	11	BM : BN :: BT : BX
But by Conſtruction	12	BT = MR
And, as was juſt now ſhewn,	13	BX = 2 NP
Conſequently from the three laſt Steps	14	BM : BN :: MR : 2 NP
But by the eighth Step	15	I : R :: MR : NP
Therefore from the laſt	16	I : 2 R :: MR : 2 NP
Conſequently from the fourteenth and ſixteenth Step	17	BM : BN :: I : 2 R
But from the firſt Step	18	LM : 2 LN :: I : 2 R
Therefore from the two laſt	19	BM : BN :: LM : 2 LN
And by changing the Places of the mean Terms in the laſt Step	20	BM : LM :: BN : 2 LN
And by ſquaring each Term	21	BMq : LMq :: BNq : 4 LNq
From whence by comparing the Antecedents and Conſequents with the Antecedents we have	22	BMq + LMq : BMq :: BNq + 4 LNq : BNq

But

rizon, for then the Line AI is parallel to it; and as the Sun riſes the Height of the *Bow* diminiſhes, and with it the Portion that is viſible, till it is 42 Degrees and two Minutes high; after which the *Bow* appears no more, becauſe then the Point I is above 42 Degrees below it.

The *Phænomenon*, we have been explaining, conſtitutes what is called the *primary* or *interior Bow*; there is alſo another exterior to this, whoſe Colours are much more dilute and faint, which

But becauſe the Triangle BML is right angled at M	23	$BMq + LMq = BLq$
And for the like Reaſon BNq + LNq is equal to BLq, therefore	24	$BNq + 4\,LNq = BLq + 3\,LNq$
Therefore from the three laſt Steps	25	$BLq : BMq :: BLq + 3\,LNq : BNq$
But BLN being a right-angled Triangle	26	$BNq = BLq - LNq$
Therefore from the two laſt Steps	27	$BLq : BMq :: BLq + 3\,LNq : BLq - LNq$
Now becauſe BML is a right-angled Triangle	28	$BLq = BMq + LMq$
Therefore from the two laſt Steps	29	$BLq : BMq :: BLq + 3\,LNq\ \ BMq + LMq - LNq$
And ſubſtracting the two firſt Terms, *viz.* BLq and BMq out of the two laſt Terms reſpectively, we have	30	$BLq : BMq :: 3\,LNq : LMq - LNq$
But by the firſt Step	31	$LM : LN :: I : R$
Therefore ſubſtituting I and R in the Room of LM and LN in the 30th Step, we have	32	$BLq : BMq :: 3\,Rq : Iq - Rq$

The

which for that Reaſon is called the *ſecondary* Bow. The Progreſs of the Rays of Light through the Drops of Water, in forming this, is as follows.

Let A (*Fig.* 85.) repreſent the Eye of a Spectator, SB and SC two of the Sun's Rays entering the Drops, as expreſſed in the Figure, and after being twice reflected in each Drop, *viz.* at D and D, let them paſs out, the one at E, the other at F, by which Means they will be ſeparated into their homogeneous Colours, the *Violet* and moſt refrangible Light being

The Proportion therefore which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, when Rays of any particular Colour paſs out of Air into Water, being known, the Proportion, which the *Radius* BL bears to BM, will be thereby determined; and therefore the Angle BLM, of which BM is the Sine, will alſo be known, and therefore alſo the Angles LBM, which is equal to ABQ, the Degree of Obliquity wherewith the *efficacious* Rays enter the Drop. But the Line BM being known, the Line BN may be alſo had by the 17th Step, and therefore alſo the Angle BLN of which it is the Sine, and therefore the Angle LBN too, or its Equal LEN; and therefore alſo the Complement of this laſt to two right ones, *viz.* KEB. If now we ſubſtract the Angle LBN out of LBM, we gain the Angle EBK, and conſequently the third Angle in the Triangle EBK may be from hence known, *viz.* the Angle BKE. Now if we double this, we have the Angle AKI, which was the Angle ſought.

If a Computation be made after this Manner with the *Ratio* of 108 to 81 (or which is the ſame thing) that of 4 to 3, for the *Red* Rays, the Angle AKI (that is, the Angle GAI in Fig. 82.) will be found 42 Degrees and 2 Minutes; and if we uſe the Proportion of 109 to 81, which is the Proportion of Refraction in the *Violet* coloured Light, the Angle AKI (or HAI in Fig. 82.) will be 40 Degrees 17 Minutes. And the Difference between theſe two Angles (that is, the Angle GAH in Fig. 82.) will be the Breadth of the Bow.

conveyed

conveyed from the uppermost Drop to the Eye at A in the Line EV; while the *Red*, and least refrangible, is carried from the lower one in the Line FR, and the intermediate Colours from the intermediate Drops: So that in this Bow the Colours will be in an inverted Order, with respect to those of the other, the *Red* being the innermost in this, which was the outermost in that. The Colours in this will be more dilute than in that, because the Rays in this suffer two Reflections, in that but one; and the Angles, which the incident Rays in this make with the emerging ones, are larger than the like Angles in the other, as shall be determined in the next Note, *viz.* 50 Degrees 57 Minutes for the *Red*, and 54 Degrees 7 Minutes for the *Violet*; this Bow therefore is exterior to, and encompasses the former.

As to the Means by which Rays of Light become *efficacious* in the Formation of this Bow, it is exhibited in Figure 86. where AB, CD represent two parallel Rays incident with such Obliquity upon the Drop, that they shall cross each other before they reach the other Side; which that it is possible appears from what was said, with regard to the Progress of the Rays through the Drop XY in Figure 83. Let them then cross in the Point E such, that after Reflection at F and G they may become parallel, suppose in the Lines FH and GI, then from the Nature of the Circle it is plain, that after Reflection

Reflection at H and I they will croſs again, ſuppoſe at K, and after Refraction at V and W, will become parallel as at firſt. And ſuch of theſe as alſo enter ſo very near one another, that their intermediate ones may ſuffer the like Refractions and Reflections with themſelves, will be the *efficacious* ones, and exhibit the *Idea* of their own Colour, at a conſiderable Diſtance from the Drop. What the Obliquity is with which theſe muſt enter the Drops, and the Angle the emerging ones of each particular Colour will make with their incident ones, ſhall be determined in the Note below [b].

X Now

[b] The Progreſs of the *efficacious* Rays through the Drop BH (Fig. 86.) being as explained above, it is evident, becauſe the Angles of Incidence are every where equal to the Angles of Reflection, that the Lines BG, GI and IW are all equal, and therefore the Arches BG, GI, and IW are ſo too; and likewiſe, that the Lines DF, FH, and HV are equal, and therefore alſo the Arches DF, FH, and HV: It is alſo apparent, that the Arch FG is equal to HI, therefore FG is half the Difference between the Arches FH and GI, and conſequently it is half the Difference alſo between the Arches FD and GB which are reſpectively equal to theſe. Now, the whole Difference between theſe Arches is what remains when FG is taken from BD, therefore the Remainder when FG is taken from BD is double of FG, conſequently FG itſelf is but one third Part of BD; for, if when one Quantity is taken from another, the Remainder be double to the Quantity taken away, it is plain that other muſt contain the Quantity taken away three times.

Now the Rays AB and CD being ſuppoſed infinitely near one another, the curvilineal Spaces BED, and FEG may be conſider'd as ſimilar Triangles, and therefore EG is equal to a third Part of EB, conſequently N (the Point where the Perpendicular LN falls upon BG) being the middle Point of the Line BG, EN is equal to EG, and therefore alſo a third Part of EB.

If now as in the 84th Figure, the Triangles BTD and BXD be formed, as alſo the Triangle NOP, NO will be a third Part of

Now if we imagine the Lines EA and FA (Fig. 85.) to revolve about the Line AI which passes through the Eye of the Spectator and the Center of the Sun as before, and always to make the same Angles with it at A, they will describe the Surface of two Cones, in the larger of which will be situated the Drops that exhibit *Violet*, and in the lesser those which exhibit *Red*. So that this Bow also, were it to appear intire, would be a compleat Circle, and

BD, and NP a third Part of BX. Therefore resuming the former Process at the 11th Step, we may proceed as follows,

Viz.	11	BM : BN :: BT : BX
By Construction	12	BT = MR
And by what was just observ'd	13	BX = 3 NP
Therefore from the three last Steps	14	BM : BN :: MR : 3 NP
But by the eighth Step	15	I : R :: MR : NP
Therefore from the last	16	I : 3 R :: MR : 3 NP
Consequently from the 14th and 16th Steps	17	BM : BN :: I : 3 R
But by the first Step	18	LM : LN :: I : R
Therefore from the last	19	LM : 3 LN :: I : 3 R
Therefore from the 17th and 19th	20	BM : BN :: LM : 3 LN
Or by changing the Places of the mean Terms in the last Step	21	BM : LM :: BN : 3 LN
And squaring the Terms	22	BMq : LMq :: BNq : 9 LNq
Therefore by comparing the Antecedents and Consequents with the Antecedents, we have	23	BMq + LMq : BMq :: BNq + 9 LNq : BNq
But by the Figure	24	BMq + LMq = BLq
And	25	BNq + 9 LNq = BLq + 8 LNq

Therefore

and the ſeveral Cones, through whoſe Surfaces

Therefore from the three laſt	26	$BLq : BMq :: BLq + 8\ LNq$ BNq
But by the Figure	27	$BNq = BLq - LNq$
Therefore from the two laſt	28	$BLq : BMq :: BLq + 8\ LNq : BLq - LNq$
But by the Figure	29	$BLq = BMq + LMq$
Therefore from the two laſt	30	$BLq : BMq :: BLq + 8\ LNq : BMq + LMq - LNq$
And ſubſtracting the two firſt Terms out of the two laſt Terms, we have	31	$BLq : BM :: 8\ LNq : LMq - LNq$
But by the firſt Step	32	$LM : LN : I : R$
Therefore	33	$BLq : BMq :: 8\ Rq : Iq - Rq$

Now the Proportion of I to R being known, the Proportion which the *Radius* BL bears to BM is had by the laſt Step. But to avoid the Confuſion which a Multiplicity of Lines may occaſion, let the 86th Figure be transferred to the 87th with as many Lines as ſhall be neceſſary, in which let AB be the incident Ray, BG the refracted one as before. Then, becauſe the Proportion between BL and BM is known, the Angle LBM may be had, which is equal to ABQ, the Meaſure of the Obliquity with which the efficacious Rays enter the Drop; and therefore alſo its Complement to two right ones SBL. And the Line BM being known, the Line BN may be had, becauſe by the 17th Step BM is to BN as I to 3 R, and therefore alſo the Angle LBN, or its Equal LGB, and conſequently BLG the remaining Angle of the Triangle BGL; but to this is equal the Angle GLH or HLV, and if theſe three be added together, and their Sum taken from four right ones, it will give the remaining Angle about the Center, *viz.* VLB, which being halved, gives the Angle SLB; but the Method of determining the Angle SBL was ſhewn before, and therefore LSB the remaining Angle of the Triangle LBS, may be had, which Angle doubled gives the Angle VSB or its Equal ASY, which is the Angle ſought.

If a Computation be made after this manner with the *Ratio* of 108 to 81 for the *Red* Rays, this Angle will be found to be 50 Degrees 57 Minutes; if with 109 to 81, for the *Violet*, it will be 54 Degrees 7 Minutes; and the Difference, *viz.* 3 Degrees 10 Minutes, will be the Breadth of the *Bow*.

the Drops as they form the Colours of it paſs, having one common *Axis* AI with thoſe in whoſe Surfaces the Drops forming the Colours of the other *Bow* were placed, this will be exterior to, and concentric with it, and will therefore ſurround it, as obſerved above.

As Rays of Light when they arrive at the Surface of a Drop of Water never all paſs out, but are in Part reflected and in part refracted, it is evident that ſome Rays will paſs out of each Drop after having ſuffered three Reflections, ſome after four, &c. theſe alſo will conſtitute *Rainbows*; but becauſe the greateſt Part of the Rays will be loſt in ſuffering ſo many Reflections, that *Rainbow*, which is made by three Reflections, is ſcarce ever ſeen, much leſs ſuch as are made by more, &c.

It is evident, that ſince the Line AI, *viz.* the common *Axis* of the Cones, on whoſe Surfaces the Colours of the *Bow* are formed, paſſes through the Eye of the Spectator, no two Perſons can ſee the ſame *Bow* at the ſame Time, or rather, that the Rainbows, ſeen by two Perſons at the ſame Time, are formed in different Drops of Rain and in different Parts of the Heaven.

Accordingly, if a Perſon obſerves the Dew as it hangs upon the Graſs when the Sun ſhines, he ſhall ſee the Colours of the *Bow* in the Drops of Dew; but as he walks along, the Colours ſhall remove from Drop to Drop.

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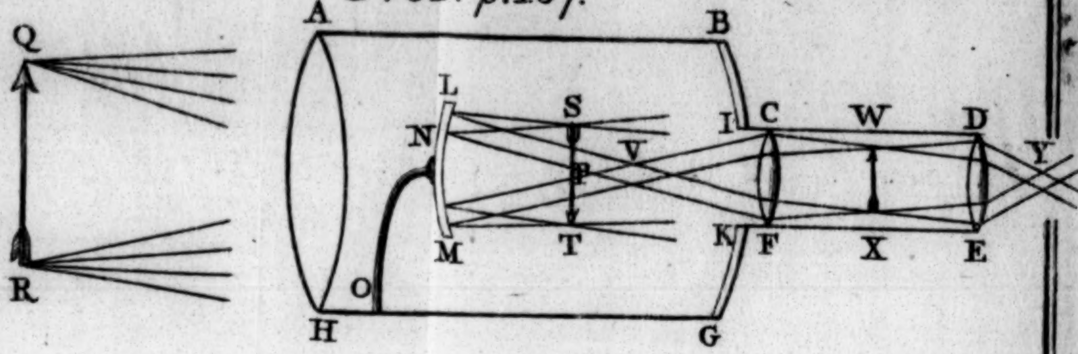
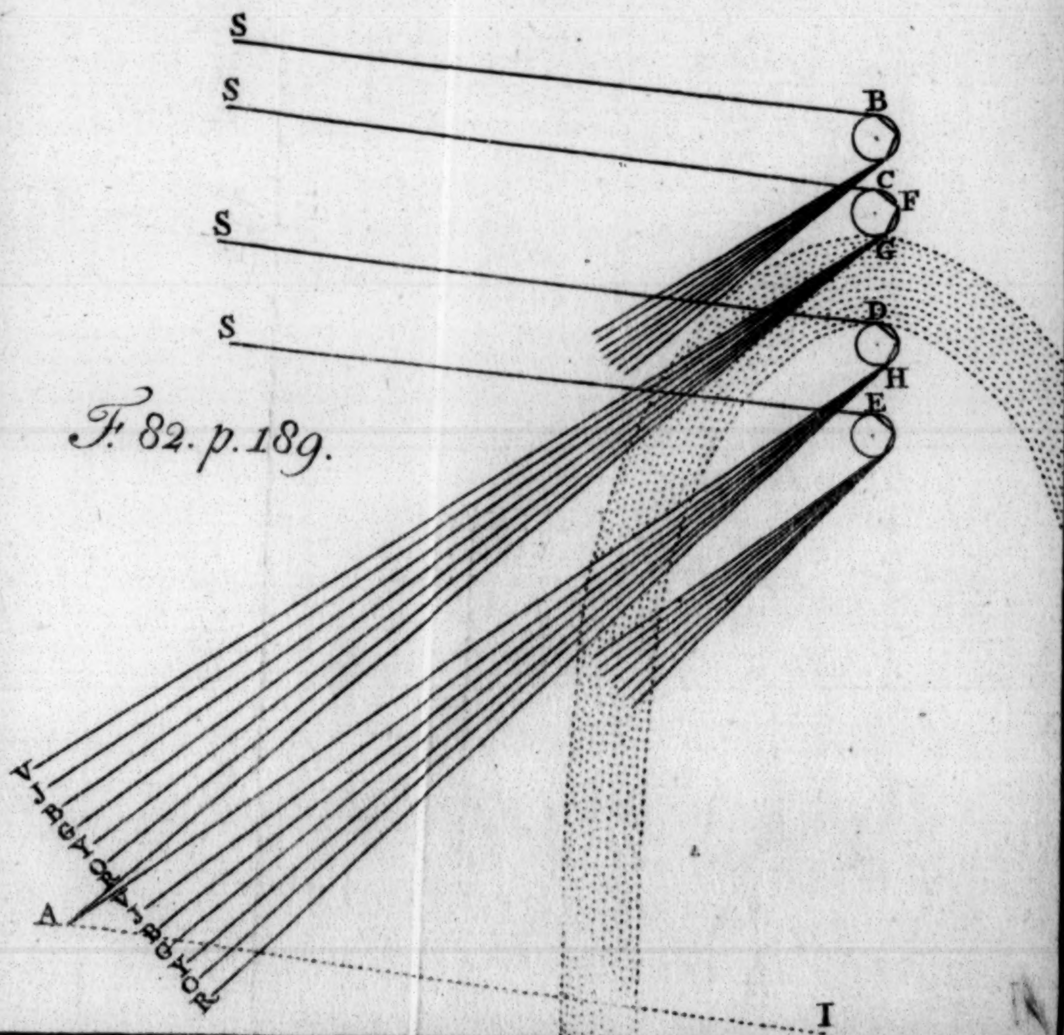
Part III. Plate XIII. p. 208.

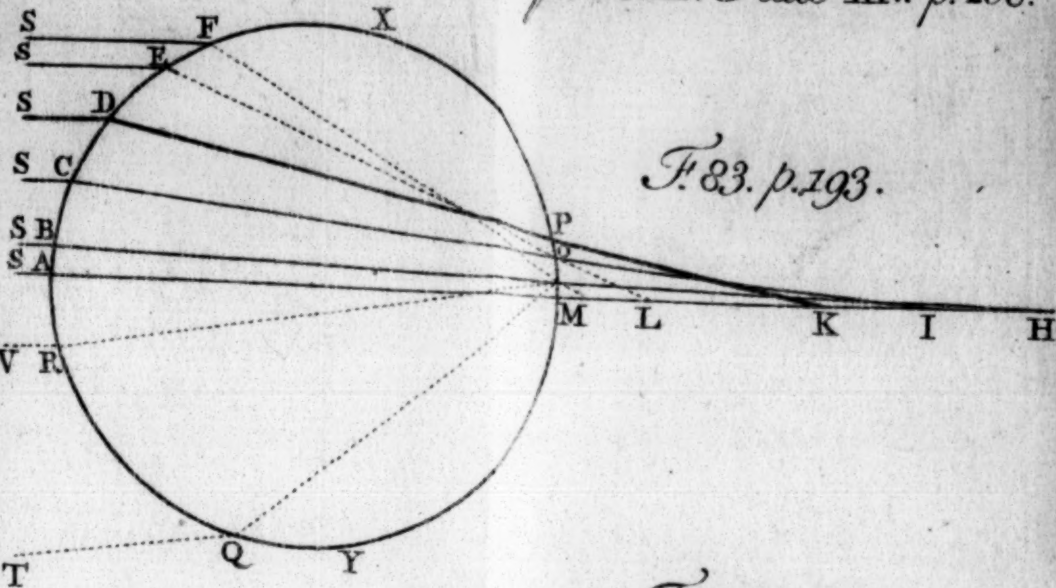
F. 81. p. 187.



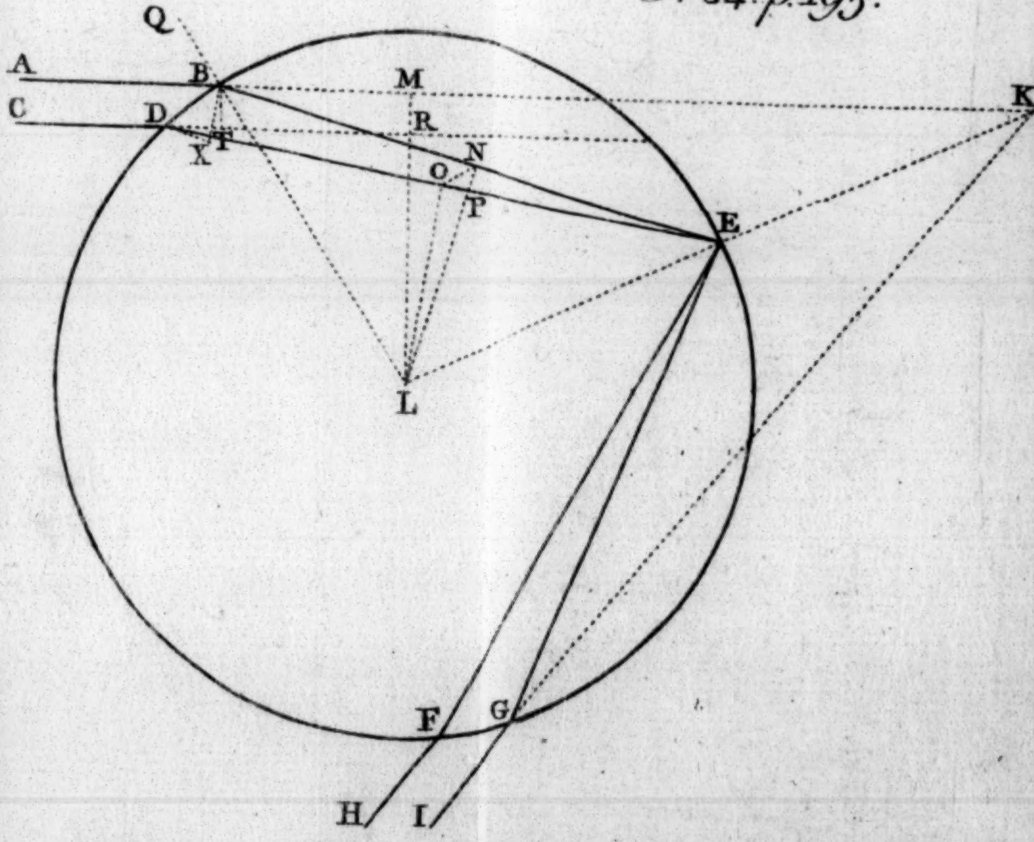


F. 82. p. 189.

Part III. Plate XIV. p. 208.



F. 83. p. 193.



F. 84. p. 195.

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Sometimes the lower Part of the *Bow* ſhall appear upon the Ground, and the upper Part of it not at all, and then it looks like a Rainbow lying along the Ground with the Extremities of its Legs turned upward into the Air: This is when the Sky is clear towards the Sun, but foggy on the oppoſite Parts, and only to a ſmall Height from the Ground.

The Moon ſometimes occaſions the Appearance of a *Rainbow* after the ſame Manner that the Sun does, but the Colours are much more faint and dilute.

And laſtly, if Water be continually thrown up into the Air oppoſite to the Sun, as from a Fountain, and there breaks into ſmall Drops, the Appearance of the Rainbow will be exhibited in them.

See more on this Subject in *Antonius de Dominis de Radiis Viſus & Lucis*; and the Authors referred to by Mr. *Johnſon*, in his *Philoſoph. Quæſt.* Chap. VII. Q. 45 & 46.

Of

Of the Obſcura Camera *and the* Magic Lanthorn.

THE *Obſcura Camera* is of two Sorts; the one is no other than a convex *Lens* fixed in an Hole in a Window-ſhutter, which *Lens*, when no other Light is permitted to enter the Room except what paſſes through it, will repreſent all the external Objects that are viſible through that Hole upon a white Paper held at the focal Diſtance of it, painted in their proper Colours. To illuſtrate this, let AB (Fig. 88.) repreſent a Window-ſhutter, CD a convex *Lens* fixed in an Hole therein, and let EF be an external Object; then will this Object emit Rays of Light of its own Colour from each Part, which paſſing through the *Lens*, as the Figure repreſents them, will be collected into Points at GH, and being there received upon a white Paper or other Surface, will repreſent the Object painted in its proper Colours, which Colours will be the ſtrongeſt of all when the Sun ſhines upon that Side of the Object that is next the Glaſs. But the Repreſentation will be *inverted*, becauſe the Pencils of Rays that flow from the Object croſs in the Middle of the Glaſs.

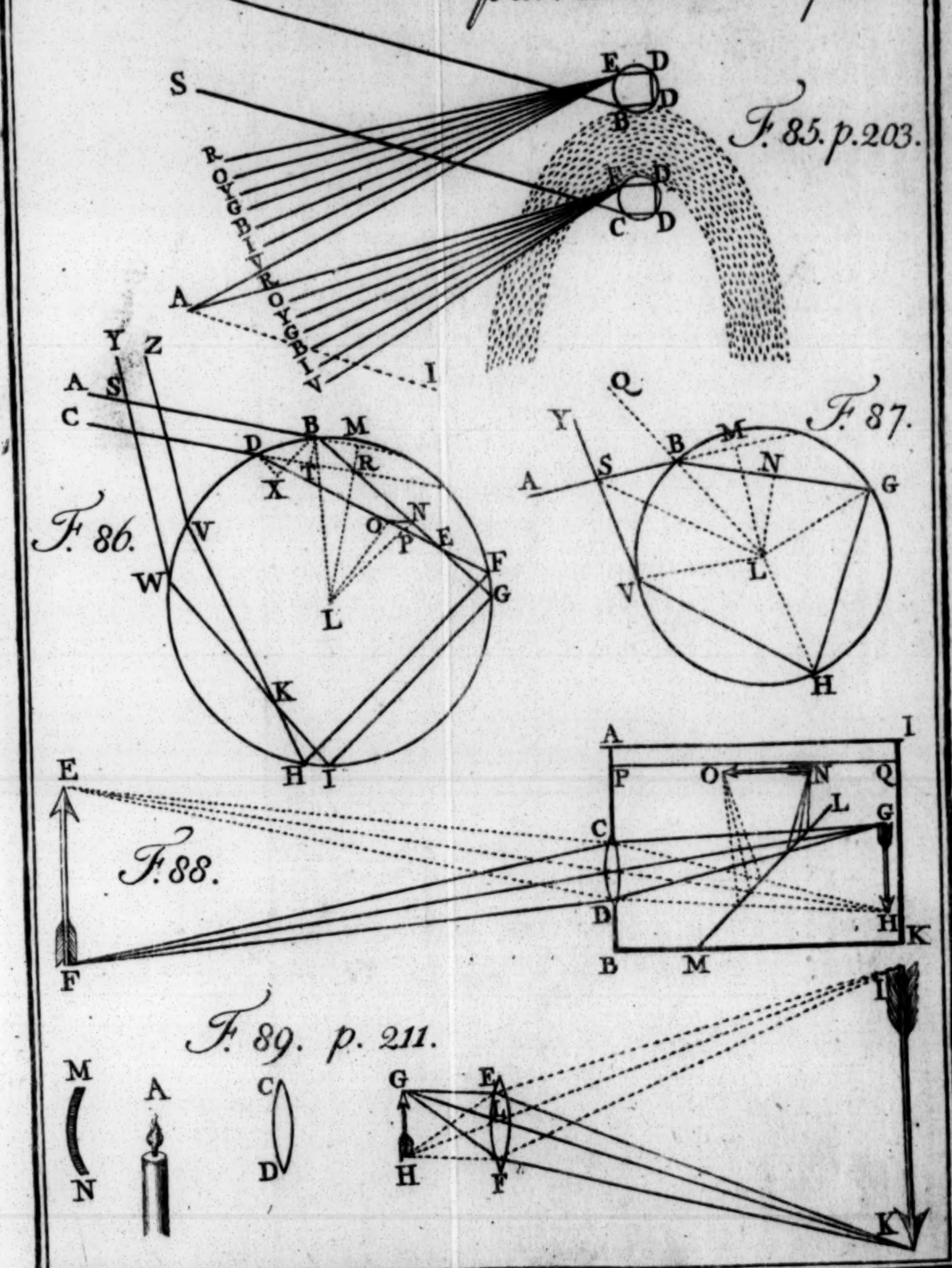
The

The other Sort of *Obſcura Camera* is that which is called the *Portable one*, and is of Uſe in drawing Pictures, taking Landſkips, *&c.* It is contrived after the following Manner, AIKB (Fig. 88.) is a Box, in an Hole in whoſe Side the *Lens* CD is fixed (or rather at the Extremity of a ſhort Tube fixed in that Hole) and in the Situation LM is fixed a Piece of Looking-glaſs making an Angle with the Side of the Box of 45 Degrees; this, receiving the Rays in their Paſſage to GH, throws them upwards, and cauſes the Repreſentation to be made in NO, which is there received upon the under Side of ſome thin Subſtance PQ, that is in a ſmall Degree tranſparent, (as thin Paper or Glaſs about half poliſhed) and ſo upon opening the Box appears ready to be drawn or copied out. But that the Colours may appear ſtrong, the Light, when the Box is opened, muſt be kept from falling upon the Paper or Glaſs, as much as may be.

The *Magic Lanthorn* is an Inſtrument invented by *Kircher*, in order to repreſent Objects much larger and more luminous than they are. It is no other than a dark Lanthorn, in the Side of which there is fixed a ſhort Tube, and in the Tube two convex *Lenſes*, and between them a tranſparent painted Image of the Object we would repreſent. The Paſſage of the Rays through the

Lenſes and the Image is thus. Let A (Fig 89.) repreſent a burning Lamp placed as in a common Lanthorn, and let CD, EF, be the two *Lenſes* placed in the Tube abovementioned, and the Picture at GH. And let the Situation of the *Lens* CD be ſuch, that the Light which falls upon it from the Lamp may be all thrown upon the Picture GH, by which Means it will be ſtrongly illuminated, and being tranſparent, will throw out Rays in Plenty the other Way: Which Rays, in paſſing through the other *Lens* EF, let us ſuppoſe to be collected into their reſpective *Foci* on an oppoſite Wall at IK, and to form an Image there. Which Image will be larger than the Picture in Proportion as the Diſtance IL is greater than LH; becauſe the Angles ILK and GLH are equal; and the Room being dark every where elſe, it will appear very bright, if the Picture be ſtrongly illuminated by the Lamp. And beſides the abovementioned *Apparatus*, there is ſometimes a concave Reflecter placed within the Lanthorn behind the Lamp, as at MN, to give a ſtronger Illumination to the painted Image at GH.

Part III. Plate XV. p. 212.



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